Teaching and Learning Primary Numeracy: Policy, Practice and Effectiveness

A review of British research for the British Educational Research Association in conjunction with the British Society for Research in the Learning of Mathematics

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EDITORS' INTRODUCTION

Mike Askew & Margaret Brown

This monograph is one in a series, each following a national seminar held as part of the BERA National Events Initiative. This initiative was launched by the BERA President, Professor Pam Lomax at the BERA Annual Conference at Queens University Belfast in August 1998, with the aim of reviewing findings from areas of British Educational Research which have significant implications for educational policy and practice.

It was suggested that Numeracy would constitute a topic of particular relevance since the National Numeracy Strategy was due to be introduced in all English primary schools in the year from September 1999 to August 2000. Related initiatives are also happening in other parts of the UK, in particular all are involved in Mathematics Year 2000, which is in turn part of a UNESCO world-wide initiative.

The BERA National Event relating to Numeracy

This took place on Saturday February 26th, 2000 at the School of Education at the University of Exeter. Although there is no BERA Special Interest Group (SIG) in mathematics education, a similar function is fulfilled by the British Society for Research in the Learning of mathematics (BSRLM) which is formally associated with BERA. BSRLM has been in existence for nearly 20 years and now works closely with BERA, for example by sponsoring symposia at the BERA Annual Conference. It was decided to organise the BERA National Event as an integral part of a BSRLM Day Conference; this would ensure a good attendance of key researchers in the field and streamline publicity. Also by fitting it into an already planned conference we were able to arrange it earlier than would otherwise have been possible, and close to the launch of the National Numeracy Strategy in September 1999.

As organisers of the BERA event, and as a basis for this resulting research review to be published by BERA, we tried to identify some key aspects of research relating to numeracy which each had clear implications for policy and practice. We then approached an expert in each area to provide a brief (< 1500 words) draft review of relevant UK research, outlining some key issues and findings, together with implications for practice. We were fortunate that every one of the people we approached agreed to undertake the task, in one case jointly with a colleague, and during the fortnight before the conference the papers steadily appeared on the BSRLM web-page.

In the event the railway system did its best to sabotage the programme, with a queue of trains arriving in Exeter from different parts of the country, including London, stuck for over an hour a few miles out of the station while a fire was extinguished in the engine of the Manchester train. Nevertheless the programme went ahead with only a few changes of programme to accommodate the late arrivals. The event was well-attended by over 80 people, including researchers from Holland and Australia.

Perhaps unsurprisingly, much of the discussion centred on recent government initiatives, including aspects of the National Numeracy Strategy, which were examined in the light of the research base. Some examples of controversial points from the papers in this review which caused debate were:

- the effect of shifting the focus of early years teaching to abstract ordinal counting rather than cardinal strategies relating to sets of objects;
- related to ordinal v. cardinal is the question of place-value or quantity value (e.g. at what stage should the 3 in 38 be identified as 3 tens rather than thirty?);
- the need for standard as opposed to effective calculating algorithms;
- the lack of convincing evidence that use of ICT is a positive factor in raising long-term standards;
- the possible reasons for poor performance of some minority ethnic groups;
- the fact that holistically based practices rooted in teachers' beliefs and/or cultures seem to be more salient than technical factors in relation to effective pedagogies;
- the effects of the reduction in the length of initial and inservice training.

The Research Review

As a result of the discussion of each paper which took place at the National Event and comments from the editors, the authors revised their papers, and it is these revised versions which are presented in this review.

There are several limitations of the review which should be noted, and which are largely due to the need to keep it within manageable bounds.

First, the review focuses on British research, and research from elsewhere is only included where it is essential to understanding the state of knowledge.

Second, the review is confined to numeracy and not with the wider aims and topics which properly belong within mathematics. There is some discussion of the changes of interpretation of the word numeracy in the first paper, but briefly numeracy is taken here in the not unproblematic sense in which it is used in the National Numeracy Strategy, i.e. to include both basic understanding and skill with numbers and number operations in the abstract, and the solution of problems in a variety of everyday contexts. There is in fact very little recent British work on the 'number sense' aspects of understanding and using numbers, so that this is not included as a separate section.

Third, the review focuses in the main on work encountered in nursery and primary schools, and is concerned largely with whole numbers and the four operations on these rather than with rational numbers like fractions and decimals, which are introduced in primary schools but also form a major part of the secondary number curriculum.

Finally, other areas which have not been addressed are detailed studies of learning in classroom contexts, attitudes, standards of attainment and international comparisons.

In spite of these omissions there is much here which is relevant to numeracy as it is and could be taught in primary schools. Each author was asked to address specifically the implications for policy and practice, and for further research.

In fact several reviewers expressed their surprise at how little exclusively British research there was, and how much we relied on the USA in particular for most of our education research in primary mathematics. This has implications for national priorities in educational research, in which numeracy has a much smaller research base than literacy.

In terms of methodology, some reviewers pointed out that computer-based literature searches in this area were a surprisingly blunt tool; in the end there was no substitute for personal knowledge and sitting in a library trawling through piles of journals!

But hopefully this review will be regarded as a first go rather than as an end-point. It needs extending into some areas which are noted above as not covered. It is clear nevertheless that we have begun to identify an agenda both for debate and for research in various aspects of numeracy.

Acknowledgements

We are am particularly grateful to the contributing authors who did all the hard work of reviewing, conceptualising and presenting the research, to the BSRLM Executive and especially to Candia Morgan and the local representatives who organised the Day Conference and incorporated the BERA Event within it so efficiently, and to Dave Pratt who looked after the web-site with great promptness.

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NUMERACY POLICY

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Introduction

When 'numeracy' first officially entered the English language as an important element of education in the Crowther Report on 16-18 Education (DES, 1959), it had a broad meaning of 'scientific literacy'. However by 1976, 'numeracy' was understood to mean the ability to employ number skills and concepts in real-life contexts (Callaghan, 1987). More recently the National Numeracy Strategy (DfEE, 1998) has shifted the interpretation to emphasise competence at abstract number skills and relations, but to also broaden to include data handling and measurement, so that there is no longer, at least at primary level, any clear distinction between numeracy and mathematics (Noss, 1997).

Meanwhile there is now a growing appreciation of the socio-cultural nature of numeracy practices in homes and workplaces which differ substantially from school-taught methods (Plunkett, 1979; Baker, 1996; Noss, 1997).

Justification cited for government action

While McIntosh (1981) demonstrated that there has rarely been a time when there has not been criticism of numeracy standards in England, more recently government intervention stems from 1976, when a Prime Minister publicly expressed a growing concern among employers about the poor numeracy standards of school-leavers (Callaghan, 1987). However the Cockcroft Committee of Inquiry which was launched soon afterwards found little evidence of dissatisfaction (DES/WO, 1982). Concern has more recently arisen over comparative evidence of poorer standards of number performance in England than in competitor industrial countries (Prais & Wagner, 1985; Reynolds & Farrell, 1996; Keys et al., 1996; Harris et al., 1997; Basic Skills Agency, 1997; DfEE, 1999). Although it has been noted that there is no relation between GDP and numeracy levels (Robinson, 1999), nevertheless it appears that for the individual in England, innumeracy is a more significant handicap to employment than is illiteracy (Parsons & Bynner, 1999).

Government interventions 1976-1995

The setting up of the Cockcroft Inquiry was the start of the first recent Government attempt to improve national numeracy standards in England and Wales. The recommendations in the Cockcroft Report (DES/WO, 1982) included a 'Foundation List', setting a minimal utilitarian numeracy curriculum, and gave greater curricular emphasis to application to real-life contexts, practical work, calculators and realistic problem-solving. This was followed up at primary level by a national project Primary Initiatives in Mathematics Education (PRIME) which included development of a Calculator-Aware Number curriculum, stressing mental mathematics but replacing standard written methods by calculator use (Shuard et al., 1991).

Government dissatisfaction with the direction of scarcely-implemented post-Cockcroft changes, together with evidence of unfavourable international comparisons, triggered the introduction of much greater prescription. This took the form of a national curriculum containing detailed teaching objectives, and national testing with publication of school results. Ernest (1991), Ball (1990) and Brown (1996) describe the clash in philosophies of education and of numeracy/mathematics which troubled policy decisions in this period; these were in fact only recent manifestations of differences which had pertained over the previous 150 years (Brown, 1999). The implementation of the reforms were not always as intended, and two revisions of the national curriculum followed swiftly upon the original implementation (Johnson & Millett, 1996).

The National Numeracy Strategy

Finally as part of a ‘Back to Basics’ agenda and citing a further round of international comparisons, the Tory Government were pressurised by Ofsted (1997) to launch a National Numeracy Project in selected inner city areas, alongside a parallel literacy project. The incoming Labour Government continued the new emphasis on basic skills by appointing a Numeracy Task Force, following an earlier initiative on literacy.

Since the National Numeracy Project had produced promising early results (Ofsted, 1998; Minnis et al., 1999), this was adopted as the heart of a National Numeracy Strategy to be implemented in most primary schools in 1999/2000, after a large-scale national training programme (DfEE, 1998). The Strategy emphasises mental calculation and combines considerable further, technically non-statutory, prescription not only of the content and scheduling of teaching but also of pedagogy and lesson-structure.

Although the Strategy claims to be research-based there are some doubts about the validity of the claim (Brown et al., 1998; Thompson, 2000). Certainly there are still some tensions between the more traditional aspects of the Strategy and the modernising tendency of the New Labour Government (Ball, 1999; Brown et al., 2000). While the Strategy is not without criticism (Hughes, 1999), it has generally been well-received in schools (Merttens, 1999).

Implications

The literature suggests that although recent government initiatives have been generally well-intentioned responses to specific concerns, the pressures of time on policy have often led to detailed implementations which are not fully thought-through, and piloted either insufficiently or not at all. Hence they result in new unanticipated problems. Attempts either to correct old initiatives or launch new ones ensue, leading to initiative-fatigue and demoralisation of teachers. In several cases new initiatives have been launched before waiting for the results of development work. Impatience with implementation which is thought to be unfaithful to the original aims has led to increasing prescription of practice, without acknowledgement of the influence on practice of the beliefs, attitudes and knowledge of teachers.

It is clear that other countries such as Japan are much more measured in their implementation of reforms, valuing more greatly the expertise and views of teachers, educators and researchers and taking more time to discuss, consult, pilot and evaluate, and produce supporting materials.

While in the numeracy area there is a developing body of knowledge about policy development and implementation in the UK, this is still small in scope. More national research and dissemination of existing international work about effective reform and implementation are badly needed to inform future developments.

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BRITISH RESEARCH ON THE DEVELOPMENT OF NUMERACY CONCEPTS

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Introduction

Research on children's conceptual development in mathematics was a rich domain in the UK in the late 70s and early 80s. Much excellent work was carried out through large investigations, such as those incorporated in the Concepts in Secondary Mathematics and Science (CSMS) study and the Assessment of Performance Unit (APU), and also more detailed analyses of children's behaviour and errors, exemplified by the investigations carried out in the Shell Centre for Mathematics Education. Bell, Costello and Kuchemann (1983) published an excellent review of this work, which can still offer significant insights to teachers for the design of assessments and lessons and the interpretation of children's difficulties. Although research on conceptual development has continued to flourish outside the UK, significantly less has been done here in the last decade. I propose here to point out some of the possible reasons for this decrease in productivity on conceptual development, review the contributions from the last decade, and consider their actual and potential impact on teaching as well as a possible agenda for the future.

Contemporary interest in children's understanding

The surge of studies on children's conceptual development in mathematics education was undoubtedly connected to constructivism. Piaget's early investigations concerning children's reasoning about number, space and geometry, fractions, proportionality, functions, and probabilities, amongst others, provoked considerable interest in the difference between two types of knowledge in mathematics which later became known as 'procedural' and 'conceptual' knowledge (Anderson, 1983) or knowing how and knowing when to use computations. Researchers in mathematics education turned in the 1970s to the need to investigate children's conceptual understanding, having become aware that much teaching and assessment in schools dealt mostly with procedural knowledge.

The interest in understanding was strengthened even more by a variety of demonstrations that many procedures learned in the mathematics classroom were often not used outside the classroom to solve problems (Hart, 1981) , were subject to bugs in their implementation (Assessment of Performance Unit, 1991; Hart, 1981), and often forgotten in adulthood (Cockcroft, 1982). Children's misconceptions were described (Hart, 1981; Bell et al., 1983) and assessments developed to help teachers identify such misconceptions. The evidence seemed to point unambiguously in one direction: more research was needed to clarify the processes involved in promoting conceptual development so that teachers could become agents in this process.

However, studies about conceptual development have become more scarce since, in spite of the fact that many books written about mathematics for primary teachers continue to emphasise the role of understanding for children’s mathematics learning (Haylock & Cockburn, 1997; Thompson, 1997). Why should this be so?

Amongst other reasons, there are at least two that can be traced through the literature. The first one seems to be a consequence of what I will propose is a misconception developed by many researchers in this domain. Research in the late 70s and early 80s documented in a variety of ways the impact of social and cultural situations as well as affective and emotional factors on children’s performance in logico-mathematical tasks. It was shown, for example, that the social context of mathematics classrooms is such that children come to treat problem solving in the classroom as a matter of implementing computations rather than thinking: they will carry out computations even if the problem could not conceivably be solved through these (Brousseau, 1997; Greer, 1997).

It was also shown that children’s performance in the classroom might be considerably worse than outside it (Nunes, Schliemann, & Carraher, 1993; De Abreu, 1994). This finding led to the questioning of whether children’s errors were the result of mathematical misconceptions or actually the result of social misunderstandings. Some researchers seem to have concluded from studies such as these that cognitive factors do not matter for children’s mathematical performance: only socio-cultural and affective factors should be considered in the education of pre-school and primary schools children. These conclusions are not justified by the studies on which they are based.

The second probable reason for the scarcity of research on conceptual development seems to be a consequence of the contributions of past research. In particular in the domain of additive reasoning, there seems to be an impression that we already know what needs to be known to promote children’s understanding, and now we can get on with the business of teaching them. The classification schemas used in research to characterise problems that can be used to assess progress in additive reasoning have become so incorporated in the teaching of primary teachers (Haylock & Cockburn, 1997; Thompson, 1997) that some authors treat them as essential ‘mathematics for primary teachers’. (Actually the classification of situations that can be solved through addition and subtraction is not a topic in mathematics but in the development of children’s understanding of addition and subtraction.) Unfortunately, although it is true that research on additive reasoning has contributed substantially to an understanding of how teachers can become agents in the process of development, it is also true that there are many gaps in the literature and that teachers per force will continue to do much teaching without the proper knowledge base.

Contributions from the last decade: additive reasoning

Research in the last decade has contributed to our understanding of several aspects of conceptual development in the primary years.

First, researchers have investigated children’s understanding of properties of operations (sometimes referred to as principles or rules). With respect to development in the domain of additive reasoning, these investigations have shown that:

- children’s understanding of commutativity of addition is a relatively early development but should not be taken for granted in the infant classroom (Bryant et al 1999; Cowan, Foster, & Al-Zubaidi, 1993); it has also been shown that this understanding is related to the use of more efficient computation strategies (Bryant et al 1999; Cowan et al., 1993) and that it varies across problem types (George, 1992);

- children's understanding of additive composition is necessary for understanding place-value representation and also cannot be taken for granted in the infant classroom (Nunes & Bryant, 1996); the likely developmental path for this conceptual knowledge has also been identified (Nunes & Bryant, 1996);
- children's understanding of the inverse relation between addition and subtraction and of decomposition are closely related but these two are not related to knowledge of number facts (Bryant et al, 1999);
- there is evidence on the existence of implicit and explicit knowledge of properties of operations (Bryant et al 1999) but we do not know whether these two forms of knowledge can be used in the same way in the classroom or not;
- there is evidence that 8-year olds have some implicit knowledge of negative numbers but perform significantly worse if they are asked to make this knowledge explicit before solving problems (Borba & Nunes, 2000);
- it is possible to improve children's performance on tasks requiring the use of decomposition by giving them the opportunity to practice it in a familiar context where the same total is often obtained through the combination of elements with different values i.e., through counting money (Wood et al., 1998).

Children's understanding of the properties of additive operations is likely to be related to the types of strategies that they use when calculating mentally but so far evidence is only available with respect to commutativity. However, it is urgent that we find out more about the connection between the conceptual knowledge of properties of operations and the development of mental calculation strategies as these have now become a more important topic for teaching in primary school. It seems currently to be assumed that the teaching of mental calculation promotes understanding but it is just as likely that calculating strategies result from understanding. If the latter rather than the former hypothesis is correct, depending on how teaching is carried out, the teaching of mental calculation strategies may end up facing the same difficulties that were faced by the teaching of written calculation strategies.

Contributions from the last decade: multiplicative reasoning

The research on children's understanding of multiplicative reasoning has so far had less impact on teaching than the research on additive reasoning. Classifications of multiplicative reasoning problems do not have the same privileged treatment in the teaching of primary teachers, where the difficulties of multiplicative reasoning are often ignored. Research has identified the common misconception that multiplication makes bigger and division makes smaller and provided evidence that this misconception is likely to be connected to the concept of multiplication as repeated addition and division as repeated subtraction. Nevertheless, teachers continue to be encouraged to use these very ideas in teaching.

Progress in the investigation of multiplicative reasoning includes the following:

- children's understanding of commutativity of multiplication is a later development than commutativity of addition and is also influenced by problem type (Nunes & Bryant, 1996);
- children's understanding of distributivity is also a late development (Nunes & Bryant, 1996);

- children in infant classes already show some basic knowledge of multiplication and division (Bryant, Morgado, & Nunes, 1992); Nunes et al., 1993) with the understanding of the inverse relation between the divisor and the quotient in division lagging behind the ability to solve sums with the support of manipulative materials (Bryant et al., 1992);
- children are able to use their understanding of multiplication to solve division questions much earlier than they are able to think of using division strategies to solve multiplication problems (Nunes et al., 1993);
- children's understanding of inverse relations when considering multiplicative relationships appears much later than expected by some mathematics educators in the past little contextual variation in performance in multiplicative reasoning tasks has been found but significant effects of the mathematical terminology used were documented (Nunes et al., 1993).

Potential impact of the research on conceptual development and a research agenda for the future

As pointed out, research on the development of additive reasoning as assessed by children's performance in problems of different types has become incorporated in textbooks for teaching primary teachers and may already have an impact on the design of lessons and assessments. What is not clear is to what extent teachers may be able to support children's learning beyond providing them with a variety of problems and discussing variations in the solutions provided by children. Although there is some research on possible developmental mechanisms, this research so far has not been considered in teacher education texts.

The research on children's understanding of multiplicative reasoning and the properties of the four operations does not seem to have had a noticeable impact yet although its relevance to teaching is undeniable. The current focus on the teaching of mental calculation without considering how the moves made in mental calculation relate to the children's understanding of the properties of operations is cause for concern. Recommendations to teach multiplication as repeated addition and division as repeated subtraction are also cause for concern. Research suggests that such teaching may be at the root of later misconceptions. Alternative models for teaching have been shown more effective in experimental studies (Clark & Nunes, 1998) but evidence is still limited and more research is urgently needed.

In view of the present emphasis on mental calculation, research that examines the significance of co-ordinating instruction on mental strategies with instruction on properties of operations is urgently needed. Considering the misconceptions shown by children about multiplication and division and negative numbers, further research on these domains is also necessary.

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BRITISH RESEARCH ON MENTAL AND WRITTEN CALCULATION METHODS FOR ADDITION AND SUBTRACTION

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Introduction

Early research studies on mental calculation were quantitative in nature and often employed the technique of chronometric analysis. This involved taking measurements of the response times of young children involved in calculating simple sums and differences and then inferring the nature of the strategies used from these measurements. Given our increased knowledge of the wide range of calculation strategies used by children, the limitations of such an approach , are now evident (Threlfall et al., 1995). This more detailed understanding of mental calculation strategies has been gleaned from studies which are of a qualitative nature. The research methods involve asking children to execute a calculation in their head and then describe how they worked it out. The interviews are usually recorded on audio or video tape for later transcription. Despite the concerns expressed about issues of reliability and validity (Ruthven, 1998) this procedure is currently considered to be the most effective way of gaining access to the thought processes of children involved in mental calculation.

Mental strategies

Given the lack of a tradition of teaching mental calculation in Britain it is inevitable that research studies in this area initially focused on attempts to identify the actual methods that children use (Jones, 1975; Thompson, 1989; Aubrey, 1993). Various taxonomies have been developed for these methods, and there is general agreement that, for the operation of addition with numbers to 20 the following strategies represent increasing levels of sophistication: count all, count on from first number, count on from larger number, use known number facts and derive a number fact (Denvir & Brown, 1986; Gray, 1991; Suggate, 1995; Thompson, 1995).

There is less agreement on a taxonomy for strategies involving the addition and subtraction of numbers from 20 to 100 (Jones, 1975; Aze, 1988; Harries, 1994; Gray, 1994; Moore, 1996; DfEE, 1999; QCA, 1999; Thompson, 2000b). However, recent research argues for the following classification system: the partitioning or split method (47+36 as 40+30=70; 7+6=13; 70+13=83) and the sequencing or jump method (83-47 as 83-40=43; 43-7=36). A variation of partitioning is the mixed method (83-47 as 80-40=40; 40+3=43; 43-7=36), and an extension of sequencing is compensation (47+36 as 50+36=86: 86-3=83). A fifth strategy, complementary addition, is often used for solving difference problems. Using this procedure the difference between 83 and 47 would be calculated as: 47 to 50 (3); 50 to 80 (30); 80 to 83 (3), and the three steps 3, 30 and 3 would be added together to give 36 (Thompson & Smith, 1999).

Counting and mental calculation

There is substantial research evidence to suggest that counting should constitute the basis of the early years number curriculum (Thompson, 1994a; Aubrey, 1996; Maclellan, 1997). However, particularly in the case of lower attaining children, there is a worry that over-dependence on counting may lead to their not committing number facts to memory (Gray, 1993; Askew & Wiliam, 1995; Tacon et al., 1997). On the other hand, even some children who know many number facts and have developed a range of sophisticated calculation strategies combine these facts and strategies with counting techniques in order to derive unknown facts (Thompson, 1995). Children need to learn to compress counting procedures if they are to be in a position to make choices between strategies. Those who have succeeded in achieving this compression of counting procedures into known and derived facts will have developed a powerful tool for success in arithmetic (Gray, 1997).

This idea of compression is related to a different concept. There are at least two interpretations of an arithmetical expression such as 5+4: one triggers the use of procedures whilst the other makes use of numerical concepts and relationships. The symbolism simultaneously represents a process to do or a concept to know, and this leads to the idea of a procept: a symbol which ambiguously represents both a process and a concept (Gray & Tall, 1994). The ability to use mathematical procepts offers greater flexibility to the learner who can choose to calculate either by using a procedure or by drawing on those relationships inherent in the concept.

Imagery

Researchers investigating the mental imagery associated with the processing of number combinations have used children's verbal and written descriptions as a means of accessing this imagery (Gray & Pitta, 1996a; Gray & Pitta, 1997). The images described by lower attaining children suggest that they carry out procedures in the mind in just the same way as they would operate with tangible objects, whereas higher attaining children seem to focus on those abstractions that enable them to make choices. The dominant representations identified among the lower attaining children are associated with images which range from pictorial representations of a hand with fingers to iconic representations of tally lines, number tracks or number lines. Higher attaining children show evidence of an implicit appreciation of the information compressed into mathematical symbolism (Gray & Pitta, 1996b).

In one study, the responses of children asked to describe ‘what was in their head’ when they calculated revealed the extent to which their mental representations were influenced by the physical representations (verbal, pictorial, written or concrete) used by their teachers (Bills, 1999). The language of a teacher’s representation and the procedure associated with it provide children with a metaphor for communicating their own methods of calculation (Bills, 2000). This raises important questions about the most appropriate representations to use when teaching. Should we continue the UK tradition of offering a wide range of models, or should we focus, as they do in the Netherlands, on a few well-researched and effective models such as the empty number line (Beishuizen, 1999)?

Teaching mental strategies

Left to their own devices some children appear able to develop sophisticated mental calculation strategies (Gray, 1991; Thompson, 1992; Aubrey, 1993; Foxman & Beishuizen, 1999). However, there is a consensus of opinion that most children need to be taught a range of mental methods (Aze, 1988; Sugarman, 1997; DfEE, 1999), and there is some evidence that these can be taught. For example, a group of teachers identified the number facts that a group of low-attaining Year 3 children were confident with, and built on these to help them derive unknown number facts. These children out-performed a control group in post-intervention assessment: three times as many moved from a modelling strategy to the use of known or derived facts (Askew et al., 1997). In another study, reception and Year 1 children working with visual images based on Stern's structured number apparatus made more progress in developing relational mental calculation methods than did a control group following a conventional approach (Tacon et al., 1997).

An alternative approach to teaching specific strategies is to teach for strategies (Sugarman, 1994). This involves teaching specific skills, developing recall of facts and building awareness of important aspects of the number system and number relationships. These factors then contribute to the construction by the child of mental strategies appropriate for a given problem situation. A different four-part model (Thompson, 1999b) adds attitudes to the essential facts, skills, and understandings that need to be developed for the successful deployment of mental strategies. Children may have all manner of facts, skills and understandings at their disposal, but if they do not have the confidence to ‘have a go’ or take risks they are unlikely to use these facts and skills to generate an appropriate strategy.

The empty number line, developed in the Netherlands for supporting mental calculation, has been recommended in several official publications (DfEE, 1999; QCA, 1998; QCA, 1999). However, only one research study in England has been reported (Rousham, 1997) and even though there was some initial success, most children reverted to formal methods within two months. The empty number line would appear to be a powerful tool for supporting mental calculation, but it needs a careful introduction and structured development: it cannot just be introduced sporadically to supplement work using a different model (Beishuizen, 1999).

Close scrutiny of the mental calculation strategies used by children for the four basic operations suggests that there is no evidence of what is normally understood by place value in their methods (Ruthven, 1998; Thompson, 1999a; Thompson, 2000a). Mental calculation strategies utilise what has been described as the quantity value aspect of place value (56 seen as 50 and 6), whereas standard written calculations necessitate an understanding of the column value aspect (56 seen as 5 tens and 6 units) (Thompson, 1999c). This subtle, but important, difference has implications for teaching. Since it is now recommended that formal written algorithms are not taught until Year 4, it would seem to make sense to delay the teaching of the notoriously difficult aspect of place value that focuses on a digit's column value (Thompson, 2000c; Anghileri, 2000).

Written methods for addition and subtraction

There are many articles on the teaching of written algorithms, but the majority appear to be based on 'reflection' rather than on 'research'. A seminal article by Plunkett (1979) argued that,

> whereas mental algorithms are fleeting, iconic, holistic and not often generalisable, standard written algorithms, on the other hand, are symbolic, automatic, contracted, efficient, analytic and generalisable.

British research in the area of written calculation focuses on very young children's invention of idiosyncratic symbols and their attitude towards standard symbols (Hughes, 1986; Munn, 1994; Gifford, 1997); on older children's invented written algorithms (Thompson, 1994b); or on the identification of errors made in carrying out the standard algorithms (Ward, 1979; Brown, 1981; APU, 1980).

Using an ingenious game involving the annotating of tins to show how many bricks they contained Hughes (1986) found that young children (including some pre-schoolers) were able to represent small quantities, and that their representations were either pictographic or iconic, based on one-one correspondence. However, Munn (1994, 1997) found that those children who used their own idiosyncratic notation in the 'tins game' were not as successful as those who used conventional numerals when it came to deciding which tin had had an extra brick added.

Hughes' (1986) work with young children and bricks also showed that, despite the fact that some of them were at school and had been using the conventional addition and subtraction symbols in their exercise books, not a single child used them in response to the researcher's request to represent on paper the process of physically adding two bricks to a pile of three. The implication would appear to be that the children did not feel that these symbols were particularly relevant to the problems they had been asked to solve. Thompson (1994b, 1997) found a parallel reluctance to use standard written methods in his research with 117 Year 5 children involved in the Calculator Aware Number (CAN) Curriculum Project. Seventy-one percent of the children set out all of their calculations horizontally, with 14% using a mixture of vertical and horizontal layouts, and 85% consistently worked from left to right, with a further 4% inclined to vary the direction in which they worked. This horizontal, left to right approach is diametrically opposed to the vertical, right to left procedure needed for the standard algorithms.

Implications for practice, policy and research

Thompson and Smith's (1999) research on mental calculation with numbers from 20 to 100 has implications for the balance in the emphasis that should be given to the various mental strategies for two-digit addition and subtraction outlined in the National Numeracy Strategy's Framework for Teaching Mathematics (1999) (see also Thompson, 2000b). The Framework also describes a clear teaching progression for calculation, starting from mental methods, passing through jottings, informal written methods, formal algorithms using expanded notation, and culminating in the learning of standard algorithms. Research is urgently needed to ascertain the extent to which this seemingly logical progression is pedagogically sound. Current research would suggest that this path is not quite so clear cut.

The plethora of research on errors or 'bugs', mainly of American provenance, needs to be extended to cover the types of error made during mental calculation and in the various stages of the teaching progression described above. We also need to know whether the introduction of the Empty Number Line - in a manner very different from that advocated in the Netherlands - is proving successful. In fact, the introduction of the National Numeracy Strategy has generated a wealth of research topics for investigation, particularly in the under-researched area of mental, informal and expanded written methods of calculation.

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BRITISH RESEARCH ON MENTAL AND WRITTEN CALCULATION METHODS FOR MULTIPLICATION AND DIVISION

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Introduction

Much of the research on multiplication and division has related to analyses of the structure of the operations, and to children's performances in relation to different problem types. Difficulties have been identified with understanding (Nunes and Bryant, 1996, Anghileri, 2000b) and with the traditional algorithms, particularly for division (Anghileri, 2000a). The largest and most extensive study remains the Concepts in Secondary Mathematics and Science (CSMS) project where whole numbers computations and extensions to fractions and decimals were considered (Hart, 1981). Conclusions from this project note that even at secondary age, many children are still only 'groping towards ideas of multiplication and division' (Hart, 1981). Children's learning of multiplication and division presents an ongoing concern. A recent national survey reports 'too many difficulties with multiplication and division' (Ofsted, 2000: 6). Tests for years 3, 4 and 5, in a national sample of 300 schools, intended 'to pinpoint strengths and weaknesses in children's mathematical skills' after the introduction of the National Numeracy Strategy identify calculations using multiplication and division, both mental and written, as key weaknesses particularly in year 3 and year 5.

Categories of Problem

Different problem types have been found to influence children's mental and written strategies. Multiplication is identified with:

- repeated sets (e.g. 3 tables, each with 4 children);
- multiplicative comparison (scale factor) (e.g. 3 times as many boys as girls);
- rectangular arrays (e.g. 3 rows of 4 children);
- Cartesian product (e.g. the number of different possibilities for girl-boy pairs from 3 girls and 4 boys).

For each multiplication problem there are two division problems related to the same number triple with the either the number of groups, or the number in each group missing. These two distinct types of division are:

- measurement/grouping (quotition) (e.g. 12 children at tables of 4, how many tables?);
- sharing (partition) (e.g. 12 children at 4 tables, how many at each?) (Brown, 1981; Anghileri, 1989; Bell et al, 1989; Greer, 1992):

Each type of multiplication and division may be related to a different contextual problem and this structure will influence the difficulty of a problem as well as the size and types of numbers involved. Although repeated addition and sharing appear to be widely understood by primary aged children, and used in informal problem solving strategies, multiplicative reasoning is more complex (Nunes and Bryant, 1996) and difficulties with understanding multiplication and division persist beyond primary school (Hart, 1981).

Relating meanings to calculation strategies

Children as young as 5 years old show some understanding of equal grouping but have difficulty with other multiplicative ideas (Anghileri, 1989; Nunes and Bryant, 1996). Bryant (1997) found that children aged 5-7 have very little idea about the divisor/quotient relationship even though they share perfectly well. These early ideas, relating multiplication to 'repeated addition' and division to 'sharing', have an enduring effect and limit later interpretations. Misconceptions such as 'multiplication makes bigger' and 'division makes smaller', and that 'division is always division of the larger number by the smaller number' cause difficulties, particularly when non integer numbers are introduced (Hart, 1981; Bell et al 1984; Greer, 1988). Even with whole numbers, older children persist with very primitive methods like tallying and use of repeated addition and repeated subtraction to calculate with large numbers (Hart, 1981: Anghileri, 1999b). Gray and Tall (1994) suggest that the introduction of multiplication and division can present a 'proceptual divide' between those who can and cannot integrate these new ideas within their existing understanding.

Language and the commutative rule

Language is an important factor as different phrases will influence greatly the solution strategy. Interpreting 52 x 3 as '52 times 3' or '52 lots of 3' may lead to a less efficient calculation than '52 multiplied by 3' or '3 fifty twos' (Anghileri, 1991, 1995). Where a problem is set in context the structure of equal groups within the problems can be influential in determining what number is to be repeatedly added. Nunes, Schliemann and Carraher. (1993) showed that 9- and 10-year old school children did not easily accept the commutative rule for multiplication when they tried to solve context problems, such as calculating '14 lots of 3 dollars'. Specific teaching plays an important role in the developing the ability to use commutativity in solving multiplication problems (Nunes and Bryant, 1996). Division does not obey the commutative rule although many children will attempt to use it when they meet a problem such as 4 ÷ 8 (Newstead, 1996; Anghileri, 1998). Children need help to interpret formal mathematical phrases identifying meanings with appropriate solution strategies (Anghileri, 1995; 1996; 2000b; Newstead, 1996).

Relating mental and written methods

Effective mental approaches can involve partitioning numbers in ways that are different from the 'tens' and 'units' partition associated with place value. In a study of 9 to 11 year olds (n = 54) informal solutions for the problems 96 ÷ 4 and 96 ÷ 6 involved 'chunking' the 96 into 80 and 16, or 60 and 36, rather that 9 tens and 6 units, or even 90 and 6 (Anghileri, 2000a). Alternative written methods to the traditional algorithms are possible that use partitioning based on counting rather than place value (Beishuizen and Anghileri, 1998; Anghileri, 2000b). Distinction is made between place value in a holistic sense and a focus on the digits, for example calculating 1256 ÷ 6 is not the same as calculating 1000 ÷ 6, 200 ÷ 6, 50 ÷ 6 and 6 ÷ 6 (Anghileri, 1999a).

Questioning the role of traditional algorithms

Reservations about the need for standard algorithms, in a society that depends on technology for all important calculations, have been expressed by mathematics educators for many years (Plunkett, 1979; Noss, 1997; Thompson, 1997; Anghileri, 1998). It is argued that teaching standardised procedures for calculating encourages ‘cognitive passivity’ and ‘suspended understanding’ as they do not correspond to the way people naturally think about numbers. Studies of workplace mathematics show that pencil and paper methods used by adults are rarely those traditionally taught (DES/WO, 1982) and workplace requirements differ from skills taught in school (Harris, 1991). Recent evidence (Noss, 1997) suggests that widespread use of computers requires a workforce with sophisticated understanding of the mathematical basis of models incorporated in software, rather than traditional computational skills which are more quickly and accurately performed by machines. The National Curriculum no longer requires standard written procedures for calculating but requires ‘efficient’ methods (DfEE, 1999, annexe p11) and choice of ‘appropriate way(s) to calculate’ (p70).

Difficulties with the standard algorithms

Research studies have identified difficulties with the standard algorithms which provide efficient written methods when they are understood but often lead to errors where they are incompatible with intuitive approaches (Anghileri, 1998, 1999a). When faced with large numbers, many children continue to use inefficient counting or tallying strategies (Anghileri, 1999b; Anghileri and Beishuizen, 1998).The algorithm is also used inappropriately, for example, in National Test calculations such as 568.1 ÷ ? = 24.7 which are designed to be done using a calculator (QCA, 1998).

Children tend to use algorithms as ‘mechanical’ procedures. Where they do not understand the procedures, they are unable to reconstruct the processes involved. Ruthven and Chaplin (1998) refer to ‘the improvisation of malgorithms’ to describe children’s inappropriate adaptations of the procedures. In tests with year 5 children (n = 276) success rate on the calculation 64 ÷ 16 was 54% with successful children using the related facts that 64 ÷ 8 = 8 or that 16 + 16 = 32. Typical wrong attempts involved use of the algorithm first dividing by 10 and then by 6. Other attempted to use the algorithm resulted in the answer 61 r 2 obtained from 6 ÷ 1 = 6 and 4 ÷ 6 = 1 r 2 (Anghileri, 1999b). Remainders in division cause difficulties. In a study of 10 and 11 year olds (n = 54) only 11% were correct in calculating 34 ÷ 7, many unsuccessful answers being 5r1 (Anghileri, 1998).

Difficulties also arise with zeros in calculations with multiplication by zero presenting difficulties (Hart, 1981). A common error resulting from use of the division algorithm involves a missing zero in the answer (answering 28r12 rather than 208r12 in the solution to 1256 ÷ 6) (Anghileri, 2000a). Further research is needed to consider which written methods children can use with confidence and understanding.

Comparing English and Dutch approaches to division

A comparison of English (n = 275) and Dutch (n = 259) year 5 children for ten division problems involving 1 and 2-digit divisors found better success rates and greater improvement over a 5 month period for the Dutch children. Overall success in January was 47% (Dutch) compared with 38% (English) while in June the results for the same problems were 68% (Dutch) and 44% (English) (Anghileri, 1999b). English children used the algorithm inappropriately and with limited success. Of the

38% of the items attempted in January using the traditional algorithm, 18% were correct. In June, 49% of the items were attempted using the traditional algorithm with only half of these correct.

Dutch teaching approaches based on counting strategies with efficiency gained through 'chunking' appear to build more effectively on children's mental strategies than procedures based on place value (Anghileri, 2000a; Beishuizen and Anghileri, 1998). This procedure was equally appropriate for any size divisors.

Extension to fractions and decimals

Calculating with fractions and decimals will involve the association of meanings with the numbers and with the operations. Fractions cause considerable difficulty when formal calculating procedures are used and even at secondary level multiplication and division problems are 'successfully completed by few' (Hart 1981). Informal methods need to be encouraged in primary school with the recommendation that formal procedures for calculating with fractions are deferred until secondary school (Hart, 1987).

Complexities arise with decimals and it is suggested that '50 per cent of children ..by the time they leave school ... have a reasonable understanding ..whereas the lower 50 per cent still have considerable gaps' (Hart, 1981: 64). Recommendations suggested that emphasis is shifted from routine techniques to a focus on understanding the principles of decimal representation and computation with the help of sensible calculator use.

Implications for policy and practice

With a wealth of evidence pointing to the difficulties children have in understanding and using formal written procedures for calculating, and the expectation that children 'use mental methods if the calculations are suitable' (DfEE, 1999:69) there is a need to re-assess the primary school curriculum. Place value and rehearsed formal procedures are no longer central as flexibility is called for in matching appropriate strategies to particular problems (School Curriculum and Assessment Authority, 1997 ; Anghileri, 2000b). Children are expected to interpret problems in a meaningful way, making connections between the conceptual and calculational aspects of mathematics. By focusing on the development of number sense through encouraging mental methods and informal written strategies, children will develop confidence in their own approaches to problem solving and maintain an inclination and enthusiasm for mathematics.

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BRITISH RESEARCH ON DEVELOPING NUMERACY WITH TECHNOLOGY

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Introduction

Technology has radically changed the character of numeracy outside the school (Noss, 1997). In the workplace, numeracy is increasingly mediated by computerised systems; what characterises the numerate employee is the capacity to work effectively and critically as part of a human/machine system. Indeed, over 20 years ago, as calculators were colonising the workplace and taking tentative steps into schools, an inspector of schools proposed that basic numeracy should be redefined as 'the ability to use a four-function electronic calculator sensibly' (Girling, 1977). A series of studies of workplace mathematics, initiated soon after, reported that 'electronic calculators have dramatically reduced the use of traditional written methods for all kinds of calculations' and that computerisation was increasingly leading to the 'incorporation of calculations into the programs' (Fitzgerald, 1985). Findings of extensive use of informal mental strategies by both adults and children led to a suggestion that: 'With mental methods… as the principal means for doing simple calculations… calculators… are the sensible tool for difficult calculations, the ideal complement to mental arithmetic' (Plunkett, 1979).

The calculator-aware number curriculum

Against this background, the Calculator-Aware Number (CAN) project set out to develop an exploratory approach to the teaching of number; encouraging informal methods of mental calculation; renouncing standard written methods of column arithmetic; and providing children with unrestricted access to calculators. Favourable findings were reported when the performance of the first cohort of project children on a National Foundation for Educational Research (NFER) mathematics test was compared with that of peers in other schools (Shuard et al., 1991). For the second cohort, it seems that the margins in favour of CAN children were smaller (Foxman, 1996).

A more recent study analysed the progress of a cohort of children through neighbouring primary schools, including some previously involved in the CAN project (Ruthven et al., 1997). While children in the post-CAN schools performed no differently on average from their peers in KS1 mathematics testing, more were found at each extreme of the attainment distribution. At KS2, such differences did not persist, but another contrast was found. Children in the post-CAN schools were more liable to compute mentally, and to adopt powerful mental strategies (Ruthven, 1998).

Calculator and computer use under the National Curriculum

Although the 'calculator-aware' approach influenced the guidance accompanying the original national curriculum, little account was taken of its ramifications in developing curriculum orders and test designs, and the tone of official pronouncements became increasingly 'calculator-beware' (Ruthven, 2000). Nevertheless, calculators were widely available to older children. National surveys found the proportion of schools reporting that children had access to calculators in most or all lessons as 18% at KS1, 48% at

KS2 and 75% at KS3 (Seaborne, 1996); being used most commonly for executing routine computations and checking answers, then for solving complex problems, and less widely for developing number concepts (Keys et al., 1997).

A recent review noted that research in this area has focused predominantly on primary education (SCAA, 1997). It concluded that actual use of calculators had remained modest at this level, and any influence correspondingly limited; and that, however tempting it might be to cast the calculator as scapegoat for disappointing pupil performance, the available evidence did not support this.

Calculators touch directly on prized skills that have long been taken as fundamental components of schooled numeracy. But there has been no real redefinition to take account of the calculator; rather, its use tends still to be seen as a fallback strategy from an essentially unchanged personal numeracy, 'independent' of technology. In effect, numeracy is conceived as the 'residual' capability of an individual when technology is withdrawn; rather than as the capacity of the human/machine system.

Similarly, the computer has tended to be seen not as a tool capable of reshaping numerate thinking, but as a medium for developing a pre-technological numeracy. Not surprisingly, then, monitoring reports by the Department for Education and Employment (DfEE) (based on questionnaire surveys of information technology use in schools) and by the Office for Standards in Education (OFSTED) (summarising inspection evidence on primary and secondary education) show that the classroom use of computers remains modest. In particular, OFSTED (1998) reports that many teachers are not convinced that using technology produces sufficient benefits in terms of raising standards as they are presently defined.

Computer use and classroom learning

An early attempt to gauge the influence of classroom computer use on pupil achievement was the ImpacT study (Watson, Cox & Johnson, 1993). Matched classes, chosen as making high or low use of IT, were followed over two academic years. Case studies indicated that effective use of IT was supported where teachers understood the rationale of software and were willing to experiment with it; and where they took a process view of the subject and accepted collaborative working by children. In mathematics, the high-IT-using classes all reported using topic-specific courseware presenting problems for practice and/or investigation. Some also made use of more generic tools such as databases, spreadsheets, drawing and graphing packages, and Logo. On tests of mathematical reasoning, administered in the middle of the first year of the study, and towards the close of the second year, statistically significant differences favouring children in high-IT-using classes were found at KS2 and KS4, but not KS3; the median effect size was 0.3. However, these pupils did not make greater gains over the lengthy period between test administrations. Nor was the study designed to allow causal implications to be drawn with confidence.

Another study focused on the contribution that computer microworlds might make to the development of numeracy (Noss & Hoyles, 1996). While focusing on secondary schools, the findings are also relevant to the primary sector. A Logo programming environment was extended to provide new procedures, designed to support tasks concerned with scaling 'drawings' up and down in size, with the intention of focusing pupils' attention on issues of ratio and proportion. The associated curriculum unit was implemented with a class of KS3 pupils, already experienced with Logo, for one-and-a-half hours per week over a period of 6 weeks. The task strategies and learning trajectories of pupils were documented and analysed, providing information to guide future use of the unit. Pupils made significant gains in performance on written ratio tasks between pre- and post-test, sustained at deferred post-test; this contrasted with the stable performance of a comparison group. An important feature of this intervention was the pedagogical concern to structure learning effectively, anticipating ways in which important lines of enquiry and reasoning could be stimulated and supported through pupils' interaction with the microworld (Hoyles & Noss, 1992).

More recently, reviews commissioned by the Numeracy Task Force (Reynolds & Muijs, undated) and the Teacher Training Agency (Moseley, Higgins et al., 1999) have led to the suggestion that 'there is little hard evidence for any beneficial effects of ICT on numeracy in the primary age range' (Higgins & Muijs, 1999). In ensuing research, four out of five classroom interventions aimed at promoting effective use of technology in the teaching of numeracy produced significant short-term gains on standardised tests; although the researchers caution that such gain scores may lack validity, and should not be interpreted as simple effects of ICT use. The researchers report that teachers who favoured ICT were likely to have well-developed ICT skills and to value collaborative working, enquiry and decision making by pupils; whereas teachers who had reservations about using ICT were likely either to exercise a high degree of direction or to prefer pupils to work individually (Moseley et al. 1999).

Integrated learning systems

A substantial programme of research has examined the contribution made by integrated learning systems (ILS) to the development of numeracy (Underwood & Brown, 1997; BECTa, 1998). In the early phases of the research, the basic numeracy skills of groups of children whose curriculum incorporated use of ILS were compared with controls. In the first phase, extending over 6 months, a statistically significant effect size of 0.4 favoured the ILS group; in the second phase, lasting around 3 months, the statistically significant effect size of 0.1 was in the same direction. In the third phase, extending over a year, broader achievement measures were used, assessing broader numerate reasoning. A study at KS2 and KS3, using NFER test scores, found statistically significant, but small, effects (in opposite directions). One study of KS3 national test levels and KS4 GCSE grades found statistically significant, but small effects (which were consistently negative); another found no statistically significant effects. In all three studies, there was marked deviation from these general trends: according to type of ILS at KS2; and according to school at KS3 and KS4. The final programme report concludes that ILS have shown effectiveness in developing basic skills, but not in developing the numerate reasoning tested in public examinations. Similarly, the report concludes that ILS have shown positive effects on children's behaviour, motivation and attitudes towards the use of computers for learning, but that effects on more general attitudes to schooling and school work are neutral.

The integration of technology in pedagogy

This last example illustrates the contribution that can be made by an extended programme of research, conducted in some depth, from alternative perspectives, and using varied methods. No other aspect of ICT and numeracy has been researched so extensively. Yet, the variation found between schools and classes suggests that research now needs to move into the classroom, to explore the integration of technology with pedagogy. Other studies reviewed here have indicated related avenues meriting further investigation. Perhaps most striking is the recurring suggestion that effective use of ICT to develop broader numerate reasoning is associated with pedagogical approaches in which tasks are more open ended, activity more collaborative, and teacher support less directive.

The mechanisms behind such associations remain poorly understood. Reviewing a cross-European–but substantially British–group of case studies, Clausen (1992) notes how changes in pedagogical styles and classroom cultures 'seem somehow to have been precipitated by the introduction of computer-based activities'. Nevertheless, as she suggests, rather than there being a simple direct relationship, the pedagogical affordances of technology are mediated by the ways in which teachers and students not only assimilate its use to their already established perspectives and purposes, but accommodate perturbations arising during the course of its use. This issue of the integration of technology use into a larger pedagogical system, then, emerges as a most important one for future research.

Equally, educational perspectives and purposes –not just of teachers and students, but of parents and policy makers- emerge as significant factors shaping the implications for policy and practice which might be drawn from the body of work reviewed here. In the terms of the perspectives and purposes which predominate in current public discussion of numeracy policy and practice, one would be bound to draw largely neutral conclusions. Viewed from an alternative perspective, the situation can be expressed rather succinctly: present uses of technology do not greatly enhance a schooled numeracy which continues to prize independence from technology; and this culture acts as a critical barrier to the development of forms of technology integration within schools which mirror those emerging in the workplace.

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BRITISH RESEARCH ON TEACHING AND LEARNING NUMERACY IN THE EARLY YEARS

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Introduction

What is the maths that is learned in the early years? Our notions of what is relevant to maths learning in the early years will determine the research that we regard as relevant to the review. In this review, research on the development of two key number abilities–counting and symbol use–will be included on pragmatic and conceptual grounds. The pragmatic grounds are that these are both abilities that the later primary curriculum expects. The conceptual grounds are that recent research has underlined the early (i.e. preschool) development and the potential importance of these abilities. Children’s experiences before and out of school, early individual differences in ability, the nature of maths teaching in the early years and the early years maths programmes that are currently in use in Britain are all relevant areas that will also be covered. For the purposes of this review ‘early years’ will be defined as the period from 0 to 6 years because of the informal learning strategies that are most useful during this phase of development.

Communication in the early years

Effective mathematical communication between adults and young children requires that adults take account of differences between their own understanding and that of the children’s. In mathematics teaching generally, communication of reasoning can be problematic, and often relies on analogy or metaphor. Nunes, Light and Mason (1993) found that children’s progress in measurement involved progressive inter-subjectivity in their communication with each other. Focusing on the language of the classroom, Solomon (1998) found even in the first year of schooling a teacher-led mathematical discourse that children had to make an effort to access.

There is some research concerning what primary teachers actually do in the classroom. Hughes, Desforges and Mitchell (2000) found that teachers are very competent at developing their own strategies for helping children to apply mathematics. Askew et al. (1997) found that effective teachers made explicit connections between different operations. Children come to school with varying levels of understanding and different strategies– effective teachers intervene with the children’s strategies without replacing them entirely.

Even given the above, there is very little research into classroom interactions around early maths for obvious reasons (such research requires a high level of resourcing relative to output). However, even the limited amount of research that there is directs us to the conclusion that appropriate language and metaphor are both necessary and sufficient to help young children develop primitive concepts into conventional mathematical thought. Such analyses of mathematical communication reinforce traditional Early Years views on the style of communication between adult and child that is appropriate to Early Years teaching and the role of pretence in such communication (Isaacs, 1930; Pound, 1999).

In addition to the Early Years maths curriculum there are well-designed curricular programmes based on recent research into children’s abilities and ways of learning.

The National Numeracy Strategy is based on idea that much teaching is oral and interactive. It gives teachers a range of strategies and a 'template' for teaching different strands of numeracy on a daily basis with the whole class. It emphasises mental calculation in the early years, with no early emphasis on written calculations. The Strategy is loosely based on current research and theory but does not make explicit links with the research literature.

There are also programmes that are aimed specifically at low-achieving children. For example, 'Paired Numeracy' (Topping and Bamford 1998) is an approach that is derived from similar approaches in reading and science that are designed to help low-attaining or failing children. It is based on current research into the effective contexts of learning. The system uses maths games in paired (co-operative) learning and peer-tutoring.

Young children's abilities

Children's ability to benefit from the school maths curriculum is influenced by their experience of maths and number in the years before they go to school. Even young babies are surrounded by a discourse of numbers and it is thought that the earliest individual differences in number ability are related to interactive experiences around number. Children's early experiences of maths are much more discursive if they attend a nursery school managed by a teacher. Their home experiences are also influenced by parents' familiarity with mathematical discourse. Active intervention can support parents in helping children at home– a negative school attitude to parents can get in the way of developing an ethos that is truly supportive . The research on experience out of school shows that families are an important source of variance in ability, but that it is not always easy to influence what happens within them.

Aubrey (1997) has investigated what children know about number on entry to school. She found that number knowledge was related to rote counting ability; children who performed well on rote counting were well on the way to level 1 of the National Curriculum. Children from low socio-economic-status families had lower scores but did make better progress. The children's abilities were often at a higher level than the demands of the National Curriculum. Her conclusion was that children's rich experience of number was often ignored at school entry.

Researchers in other countries have come to the same conclusion about the relation between children's pre-existing knowledge and the school curriculum, for example Irwin (1996) and Young-Loveridge (1989) in New Zealand and Wright (1994) in Australia (who claims that ignoring individual differences at school entry may result in maths failure at later stages).

Advances in theoretical work on early maths (e.g. Gelman and Gallistel, 1978; Fuson, 1988) have re-instated the role of counting in the early years maths curriculum. Mathematical understanding is no longer seen as developing from an understanding of sorting and matching, and it is now understood that the social functions of counting play quite a large role in the initial acquisition of a 'number string'. This early rote counting, which used to be discounted as lacking logic, is what Aubrey (1997) found to correlate with number knowledge on school entry. While such counting has no causal connection with later number logic (such as the ability to add and subtract) it does play an important role in allowing children initial entry to the discourse of number.

A number of British researchers have worked out the detailed implications for the primary curriculum of the renewed importance of counting. Thompson (1997a) has produced a classification of number strategies and their relation to counting. (counting up and down, doubling, bridging up and down, step counting, regrouping). He shows how number knowledge is based on a network of strategies that include both counting and conceptual advances on counting. Those children who don't move on from counting are disadvantaged and require intervention. Such ideas form the basis of early years mental maths.

Gray (1997) points out that counting has a positive role early in development, but needs to be replaced quite rapidly by more sophisticated cognitive concepts. Children need to recognize symbolic ambiguity (i.e. number as counting process or as a thing). A gradual compression of counting allows children to use numbers symbolically (e.g. count-on strategy). Cowan & Ioakimidou (1999) found several preschool children who expected addition to be commutative even though they were unable to add and uncertain about the importance of order in verbal counting. This suggests that understanding of commutativity is not based on addition skill or on reflection on counting.

The Early Years maths curriculum

These findings all have relevance for the teaching strategies based on counting and mental number that are now thought appropriate in the early years. Early counting is more important than previously thought, but it is important to make sure that children move on from counting strategies as soon as they are able. It is also important to make sure that mathematical discussion is not limited by the children's apparent lack of understanding of number logic, since such discussion is the main source of their learning.

The findings on counting have considerable relevance for the role that written numerals play in the early years curriculum. Thompson (1997b) has pointed out that place value is a concept relevant only to written numerals, that it is not required for mental facility with numbers, and is logically not required in the initial curriculum if early years teaching concentrates on mental maths ability. Hughes (1986) showed that learning of number symbols is often procedural rather than conceptual ñ even in high ability children. Munn (1995) showed that early use of number symbols in problem solving was closely related to the development of number concepts. Tolchinsky Landsmann and Karmillof-Smith (1999) claim that young children distinguish different domains of symbol knowledge (writing, numbers, drawing) before they can accurately produce domain-relevant symbols, but that this distinction is not apparent in their use of notation as a communicative tool. Dockrell & Teubal (in press) claim that it is only when children learn the unique referential function of particular symbols that children can manipulate them or use them in problem solving. There is a considerable gap in our knowledge of just how young children develop the ability to use number symbols in problem solving and this is a growing research field.

Implications

In recent years there has been a revolution in our understanding of early maths learning, based on a revised assessment of the role of counting. This has by and large been reflected in changes in early maths curricula around the world. It is apparent that the small amount of research on teacher practice does not yet match the volume of research into children's understanding of number. There is a pressing need for further research into classroom discourse processes in the early years and for the results of this research to be related directly to maths teaching in the early years. There are a number of systems in use in Britain for teaching, supporting and remediating maths in the early years that are based in some way on current research. We need to link current research into early years maths more closely to classroom practices and to particular programmes. There is a particular need for more research into the development of children's use and understanding of written numerals.

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BRITISH RESEARCH INTO SCHOOL NUMERACY IN RELATION TO HOME CULTURES

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Introduction

This paper is a review of research into the understanding of the impact of children's home cultures on their achievement in school mathematics. In the international context, interest in this area derives from a recognition of the socio-cultural nature of mathematical practices. Initial empirical support for this view emerged from the analysis of the mathematical practices of social and cultural groups with limited or no participation in Western schooling. Examples of this research can be found in the fields of ethnomathematics (Ascher & Ascher, 1980, Bishop, 1988a, Bishop, 1988b, D'Ambrosio, 1985), anthropology (Lave, 1988) and developmental psychology (Gay & Cole, 1967, Nunes et al., 1993, Saxe, 1991).

In the 1980's reports on the mathematical achievements of populations living in highly schooled societies started to cause some disquiet. In Britain, the Cockcroft report (1982) stressed discrepancies of performance between distinct social groups (e.g. boys and girls) and between social practices (e.g. school and work). This led researchers to argue that a socio-cultural approach to mathematics learning was necessary in order to address issues related to participation in multiple social practices within a society. Examples of such research in Britain include (Harris, 1991, Hoyles et al., in press, Noss, 1997, Noss et al., 1999, Solomon, 1989, Walkerdine, 1988).

Research adopting a social-practice approach to children's home-school mathematical learning in Britain is still in its early stages (Abreu & Cline, 1998, Baker et al., 2000). Nevertheless, findings from other countries such as Brazil, Portugal (Abreu, 1995, Abreu et al., 1997) and the USA (Brenner, 1998, Civil, 1998, Civil & Andrade, in press, Masinglia et al., 1996) and also from studies on home-school literacy practices (Tizard & Hughes, 1984) suggest that it is an approach worth following. Below I attempt to address two questions: (1) What type of evidence supports the notion that there is a relationship between the home cultures and school numeracy? (2) What are the mediators in the relationship between the home cultures and achievement in school numeracy? The National Numeracy Strategy emphasises the raising of standards for all groups and suggests that engagement of parents and communities play a part in this process. In view of this the research basis to answer the above questions need to be clarified (Brown et al., 1998, DfEE, 1998).

Home cultures and attainment in school numeracy

In Britain support for the notion of a link between home cultures and school numeracy can be found in quantitative and qualitative analyses of the relationships between school achievement and students' social class and ethnic group membership. The relative achievement of students from different ethnic groups has been the subject of "The Committee of Inquiry into the Education of Children from Ethnic Minority Groups" (Swan, 1985). Gillborn and Gipps (1996) followed this report a decade later with an Office for Standards in Education (Ofsted) commissioned review of the school experiences of ethnic minorities.

They found some positive changes, such as, the "improving levels of attainment among ethnic groups in many areas of the country". They also raised concerns about issues, such as that of the growing gap "between the highest and lowest achieving ethnic groups in many LEAs" and the fact that "even when differences in qualifications, social class and gender are taken into account, ethnic groups do not enjoy equal chances of success in their applications to enter university" (p. 78). A recent Ofsted report (1999) confirms that the attainment of minority ethnic groups as a whole is improving, but some groups are continuing to underachieve.

Although these data provide a rough indication of some type of relationship between home group membership and school achievement, it is difficult to obtain a clear picture. Difficulties emerge from the:

- Categories used to attribute home-group membership - There are variations both between researchers themselves and between schools in the classification of ethnic minority groups. For instance, Gillborn and Gipps use ethnic minority "as a general label for all people who would not define themselves as 'white' in terms of their ethnic identity" (1996, p.8). This means that for instance their analysis left out any consideration of issues of relative performance among children of non-English white European background. Difficulties regarding the definition of social-class are also reported in the literature (e.g. Cooper & Dunne, 1998).
- Data held by the schools - Although government emphasis on National Testing and on the publication of school results has already generated some harmony on the data made available by schools, this is not yet the case in ethnic groupings. The 1999 Ofsted report on ethnic minority pupils mentioned that "There was considerable variation in the form and extent of the data held by the schools on the attainment of pupils from different ethnic groups. Some groups had no data available, in others the ethnic categories were crude (e.g. Asian), while others failed to analyse the performance of boys and girls separately." (Paragraph 26).

Some of the reports do not give a detailed analysis subject by subject. Since, however, mathematics is a core subject in the national curriculum, one can infer that the general trends in the groups also apply to mathematics. Information from Local Education Authorities in some multicultural areas confirms that the trend does indeed apply to mathematics (Luton Education Authority, 1998; Lambeth Education Authority, 1999-2000; Rasekoala, 1997). Research in the USA confirms similar trends as those found in Britain, i.e., it shows that the gaps between some ethnic groups have been closing in recent years, but also that the attainment of some ethnic groups remains a matter of concern (Secada, 1992; Tate, 1997). In addition, this research suggests that ethnic differences still persist even when variables such as social class are kept constant (Tate, 1997).

Data from studies such as the above offer some indication of which groups are likely to be more or less successful in their learning of school mathematics. Yet, it does not clarify the precise reasons why they perform differently. The complexity in this research is increased by observations that group membership in isolation cannot be taken as a predictor of school performance. Social-practice theory attempts to explore these differences through in-depth analysis of social and cultural mediation.

Mediators between home cultures and school

Recent studies in Britain on how the home cultures mediate the children's performance in school numeracy have considered between-group and within-group differences. Between-group differences have been explored in two particular types of studies. One type has as its starting point what children actually do in school numeracy tests. The other type has as its starting point ethnographic analysis of school and home numeracy practices.

Investigations on how performance in national numeracy tests can be affected by children's social class background have been conducted by Cooper (Cooper, 1994, Cooper, 1998, Cooper & Dunne, 1998). Theoretically they are based on the established European sociological theories of Bernstein and Bourdieu. Through a combination of quantitative and qualitative methods Cooper and his colleagues have shown that "working and intermediate class children seem to be more predisposed than service class children, at age 11, to employ initially their everyday knowledge in answering mathematics test items and that this can lead to under-estimation of their actual capacities with respect to the demands of the school discipline of mathematics as it is currently defined". Interview data shows that the children's difficulties arose from confusion about what knowledge was required in a specific context. For instance, in a "tennis item" children needed to understand that knowledge that was appropriate in a sports context was not appropriate (or legitimate) for solving a school mathematical test.

A difficulty with this research is that it does not describe the particular classroom practices to which the children were exposed. Therefore, it is unclear whether what children perceive as legitimate knowledge can be changed. For example, if classroom practices provide opportunities to externalise and negotiate conflicts, will this help the children re-define boundaries when applying knowledge in different contexts? Further research exploring relationships between children's socio-cultural backgrounds, classroom practices and their performance is necessary.

Research which is based upon an ethnographic account of school and home numeracies draws on cultural approaches to mathematics learning which have emerged from the cross-fertilisation of ideas from anthropology, psychology and ethnomathematics. Detailed analysis of organisation of practices is used to highlight potential areas of conflict between the child's experiences at home and at school (Jones, 1998). Between-group variations are often conceptualised as sources of "cultural conflicts", which, as Jones has pointed out can arise from differences in the ways parents and teachers: (1) view the parents' role in the education process; (2) define what is considered to be adequate behaviour for children; (3) structure the numeracy practices for the children.

In my own work I have been trying to expand this type of research to explore within-group variation. My starting point is that within any group there are always some children who do better than others. Taking this into consideration I have been using a research methodology where children from the same home group, but with different levels of school performance in mathematics are selected as case studies. The method was initially developed in Brazil and Portugal (Abreu, 1995, Abreu et al., 1997), and recently in collaboration with British colleagues was used to investigate home-school numeracies in multiethnic primary schools in Britain (Abreu & Cline, 1998, Abreu et al., in press). Following Vygotsky the learning and uses of mathematics were explored in terms of the mediating role of mathematical tools available in each practice (Nunes & Bryant, 1996). It also drew on European social representations and social identity theories (Abreu, 1999). This enabled us to explore the impact on uses, transmission and learning, of the way groups and individuals valorise their practices. Our research in Britain strongly indicated that the way families structure home practices to support their children's mathematical learning was linked not only to their own cultural heritage, but to their representations of what was "worthy" to be transmitted to the child. The basis of these representations can be a problem in a context where several parents reported difficulty in getting direct access to the child's school numeracy practices (this was greater in the case of ethnic minority parents).

Implications

To sum up, research in Britain into the social practices of home numeracies and how these relate to the child's transition to school numeracies is still recent but gaining in importance. The preliminary results are however encouraging and seem to be of extreme relevance for educational practice and policy.

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BRITISH RESEARCH INTO PEDAGOGY

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Introduction

Reviewing the research into pedagogy has to start with a definition of the term: a far from straightforward task. Some writers work with a model of pedagogy that examines teaching in broad terms: - grouping, layout of room, use of resources, such as the board - (Good & Brophy, 1997). Such a view would be close to what McEwan (1989), discussing the work of Komisar (1968) describes as the 'enterprise' level of teaching. In contrast to this McEwan suggests that teaching at the level of 'acts' also needs to be examined, teaching 'acts' referring to the process of actually teaching something to someone at the level of detail. As such, acts would seem to be what some researchers refer to as didactics, while others would include them within the general heading of pedagogy.

This broadening of the meaning of pedagogy to include some elements of didactics can be traced in the development of research in this area. Studies from the 1970s and 80s tend to concentrate on pedagogy at the enterprise level while more recent research has turned to examining the teaching of particular subject areas and even particular topics within those areas. This review therefore broadly follows this development from the view of pedagogy as 'enterprise' to the more particular in terms of mathematics as a discipline. However, the teaching of particular aspects of numeracy, for example written methods of calculation, are dealt with elsewhere in this review.

Teaching styles: Large scale studies

Alexander (1999a) suggests that pedagogy as a major focus for educational research is a relatively late development. In England there were a set of generic large scale, quantitatively oriented, studies of primary practice influenced by the 'traditional' versus 'progressive' debate. Both Bennett (Aitken, Bennett, & Hesketh, 1981; Bennett, 1976) and the ORACLE (Observational Research and Classroom Learning Evaluation) study (Galton & Simon, 1980; Galton, Simon, & Croll, 1980) attempted to cluster teachers into distinct teaching styles and to relate these to measured attainment in mathematics (and language). Mortimore et al (1988) were more concerned with the effects on performance of school management and classroom management policies.

The findings of these studies generally suggested that distinct teaching style clusters could not be easily identified. There was little co-operative work between children, with considerable individualised work despite the children being seated in groups. Integrated day and totally individualised working seemed less effective forms of organisation than those in which there were at most two different activities happening simultaneously in classrooms. The findings about whole class teaching were ambivalent; it appeared that questioning at a high cognitive level was the key factor and although there was some tendency for this to be more often combined with higher proportions of whole class teaching this was by no means always the case. Following up the ORACLE study twenty years on, Galton et al (1990) note that the attention paid to examining organisational strategies diverted attention from important differences in 'tactics' used by teachers within, rather than across, differing organisational styles.

Ethnographic studies

The large scale studies provided statistical accounts of representative classrooms and pedagogy. In contrast, small scale qualitative studies (for example Pollard, 1985; Pollard with Filer, 1995; Woods, 1990) provided insights into strategies used by teachers and children to 'juggle with their interests-at-hand in the ebb and flow of classroom life (Pollard op cit. p. 179) and in their attention to issues of power and control in the classroom are closer to looking at the 'acts' of teaching. However, they shed little light on the didactics of teaching mathematics per se.

The impact of the National Curriculum

The introduction of the National Curriculum marked a revival in large scale studies examining the effects of its implementation on teachers' practices. The PACE project (Primary Assessment, Curriculum and Experience) studied the implementation of the National Curriculum in Key Stages 1 and 2 (KS1, KS2), from the late 1980s to the mid 1990s and employed a mix of the large scale quantitative and small scale qualitative, studies, embedding detailed studies of classrooms within a larger sample. In terms of examining pedagogy, the PACE project followed in the tradition of the early large scale projects in looking at pupil organisation. In KS1 the PACE researchers suggest that while the teachers in the study still espoused a commitment to pupils having some classroom autonomy, the introduction of the National Curriculum meant that they were increasingly having to direct pupils' activities (Pollard, Broadfoot, Croll, Osborn, & Abbott, 1994). Data on styles of classroom organisation suggested that teachers were employing a 'mix' of teaching methods (Alexander, 1992) with an increased proportion of teachers from 1990 to 1992 indicating more use of 'traditional' methods, including whole class teaching.

In KS2 the PACE project noted that individual work increased as the pupils became older, with KS2 teachers spending approximately 30 per cent of time in whole class interaction and just over 50 per cent of their time working with pupils individual (Croll, 1996).

The introduction of the National Curriculum also encouraged more research attending to the teaching of particular disciplines. With respect to mathematics, the Evaluation of the National Curriculum for Mathematics (ENCM) project carried out at King's College, explored aspects of teachers' pedagogy in the light of the Orders for mathematics. Pertinent findings from the project included an examination of the extent to which teachers used commercial schemes and whether their teaching was 'scheme assisted' or 'scheme driven' (Millett & Johnson, 1996). With the introduction of 'Using and Applying Mathematics' into the curriculum part of the ENCM also examined the impact of this on teachers' pedagogy. Askew (1996) suggests that for many of the teachers in the study the introduction of this aspect into the curriculum had not actually led to major changes in practice. Instead the teachers had interpreted the orders in ways that allowed them to maintain their existing practices.

Linking practices and outcomes

While such research into the impact of the National Curriculum provided some insights into teachers' practices, the research was not evaluative in the sense of examining pedagogy in terms of effects on pupils' learning. With the increased media attention to England's standing in international league tables, research into pedagogy is beginning to try and link practice with outcomes.

However, little such research has been carried out in England and as Alexander (1999b) points out there is a heavy self referential character to much of this research. For example, work such as Creemers (1994) in Holland can be traced back to the work of Rosenshine (1983; 1987). But embedded within

Rosenshine's work is that assumption that direct instruction is at the heart of effective teaching. Recommendations for pedagogy are predicated on a behaviourist model of the teaching and on particular assumptions about what is to be taught, focusing in particular on 'basic' skills and procedures.

Such models of effective pedagogy of course appeal to policy makers because of the way in which they can be translated into lists of competencies and help develop a 'technology of teaching' (Reynolds, 1998). The Teacher Training Agency's (TTA) 'Effective Teachers of Numeracy Project' appeared to be rooted in a 'technical rationalist' (Ball, 1998) view of teaching: with the assumption that the characteristics of effective teachers and their practices could (a) be defined (b) be identified and (c) be reduced to a set of descriptive and prescriptive recommendations. This research examined pedagogy in terms of grouping and other aspects of classroom practice against pupils gains on a test of numeracy. No clear associations were identified between pupils gains and such aspects of pedagogy, although there was an association between teachers' beliefs about how best to teach numeracy and pupil gains (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997)

Setting aside the issue of whether or not such competencies can be identified and, if so, are the 'right' ones there is the question of how, having dissected pedagogy into such components, it might be re-constructed. While opportunity to learn, challenging questioning, formative feedback and so forth (for example Sammons et al (1995)) might be important factors, research has not demonstrated 'how these and other elements are reconstituted by teachers and children as coherent and successful learning encounters with a beginning, a middle and an end' ((Alexander, 1999b, p. 152).

Cross cultural studies

In an attempt to move from a parts to whole model Alexander (ibid.) develops a model of 'cross-cultural pedagogic continua' including, for example, teachers' questions on a continua from mainly closed to mainly open and views of knowledge informing lessons from codified, rule-bound and received to uncodified, negotiable and reflexive. On the basis of these continua Alexander provides examples of two paradigms of lessons, drawing on a number of lessons which although contextually very different (being drawn from across five countries) and focused on different content, share a number of characteristics in terms of their placings on the continua.

The first paradigm is very similar to the Rosenshine (1987) model but the second paradigm is very different. For example, Alexander's first paradigm is characterised, in part, by lessons being fragmented into small steps delivered at a brisk pace, while the second paradigm marks lessons as having little sense of pace with the shape and speed of the lesson emerging from events as they happen. Alexander argues that the two paradigms are extremes with many lessons combining elements from each and with considerable within country variation. Only in India and Russia was there strong consistency across lessons with both being located on the first, Rosenshine-like, paradigm.

Whitburn (2000) in her comparison of English and Japanese early years mathematics classrooms notes two major differences between English and Japanese societies with respect to children's learning. Firstly the difference in the perceptions of the relative effects of ability and effort, with perseverance highly regarded in Japan, and seen as more important than innate ability. Secondly the difference between the two countries in the balance of attention to the development of the individual as opposed to the group. Whitburn suggests that in Japan there is much more attention to working as a member of the group in ways that help everyone move forward and maximise the chance of group success.

Within such cultural norms it may be easier for Japanese teachers to work with pedagogic practices that focus upon detailed discussion of a small number of contextualised problems with a high level of peer evaluation of the problem solving approaches adopted. (Whitburn p. 257) (However such norms are less easy to reconcile with the high levels of individual practice of procedural applications also noted in Japan.) The initial intention of the National Numeracy Strategy would seem to be in line with one of Whitburn's recommendations - a reduction (removal in Whitburn's terms) of differentiated teaching (Department for Education and Employment (DfEE), 1999)

In another comparative study, Broadfoot et al (1993) suggest that the concern of English teachers to meet the needs of individuals make them more concerned than French teachers to motivate children through making the work interesting. Again, factors outside the classroom appeared to be significant in accounting for pedagogical differences, particularly with, in France, the high societal value placed on intellectual endeavour and French pupils displaying a clear distinction between 'work' and 'play'.

In the later QUEST (Quality in Educational Systems Trans-nationally) project it was hypothesised that the more formal pedagogic style of French classrooms would lead to French pupils performing better on assessment tasks based around learned formulae and procedures, while English pupils would perform better on problem solving and creative tasks. The assessment results supported these hypotheses: pupils performed best on assessments that reflected their countries' curriculum priorities (Broadfoot, 1999)

Such within country differences, Alexander suggests, can be traced to macro influences. Alexander's' work challenges the notion that large scale international comparative studies allow one to 'cherry pick' from practices and we appear to be moving into an era of research where cross cultural studies are beginning to take into account these broader societal influences.

Implications

In summary, detailed comparative studies suggest that differences in pedagogic practices are as much to do with macro influences as variation amongst individual teachers. In terms of implications for practice there is little specific to recommend. More English research needs to be carried out into mathematics pedagogy and practices and how these are influenced by both the culture of English schooling and teachers' beliefs.

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British Research into Initial and Continuing Professional Development of Teachers

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Introduction

Recent policy and prescription emanating from government bodies in relation to Initial Teacher Training (ITT) and primary mathematics teaching appears to be built upon a deficit model. Derived from interpretations of comparative international data it is sustained by a burgeoning national audit culture of league tables and targets to which the mathematical subject knowledge of ITT students will undoubtedly soon be added. This educational epidemiology monitoring the mathematical health of the nation has identified the quality of mathematics subject knowledge and understanding of students/primary teachers as a cause for concern. The prescription which, it is speculated, will guarantee a more competent and confident work force is: (a) specified and tested levels of mathematics subject knowledge for all trainees (DfEE, 1998 a); and, (b) specified mathematics content and pedagogy for all primary teachers (DfEE 1998 b). Whilst there is some evidence to support such a government agenda there are reasons for questioning the sufficiency of the account and the evidence that informs it.

The fragmentary nature of the evidence base

The paucity and fragmentary nature of the evidence from British studies which claim a specific focus on mathematics makes it difficult to make definitive research claims regarding ITT. Even in America the evidence base is widely thought to be ‘piecemeal’ and ‘not systematic’ (Eisenhart et al., 1991; Brown et al., 1990). Reviewing the literature indicates that there is perhaps a case for challenging the privileging of empirical evidence over theoretically constructed arguments: anecdotal accounts, not informed by explanatory theoretical frameworks, are in part responsible for much of the incoherence found. Certainly much existing British research is small scale and descriptions of student teachers have presented unilateral accounts of the complex teaching equation. The fragility of the evidence base regarding the effectiveness of Continuing Professional Development (CPD) programmes, especially those relating specifically to mathematics, is even more acute. Research on the effects of INSET is reported to be ‘meagre’ and lacking a ‘cumulative dimension’ (Halpin et al., 1990: 164).

The place of subject knowledge

The importance of subject knowledge is well documented and its lack is linked to less effective teaching (Wragg et al. 1989; Bennett & Turner Bisset, 1993; Simon & Brown, 1996; DES, 1983, 1988; Ofsted, 1994, Rowland et al., 1999) and over reliance on commercial schemes (Millett & Johnson, 1996). Mathematics and English primary training courses have been judged to be amongst the most satisfactory (DES, 1991). Yet, the change in subject-matter/substantive/syntactical knowledge of mathematics of PGCE student teachers (n=59) during training was found to be not significant (Carre & Ernest, 1993). Indeed, they displayed the same misconceptions as children (Bennett et al., 1993; Ball, 1990). Askew et al. (1997 a: 65; n=90), however, found that ‘more’ was not necessarily ‘better’ when they correlated teachers' mathematical knowledge, measured in terms of qualifications, against pupil learning outcomes. Recent debate regarding teacher knowledge has been stimulated by the influential work of Shulman (e.g. 1987) in the USA and concerns the need to address pedagogic content knowledge (PCK) in ITT rather than subject knowledge per se. Critics challenge the credibility of such a distinction (McNamara, 1991); and the underlying absolutist view of mathematics, and transmission view of teaching, that it presents (Meredith, 1995; Stones, 1992). Additionally, PCK is thought to be situationally and experientially grounded in, and constrained by, classroom experience; and related to knowledge, values and epistemological beliefs rather than ITT (Aubrey e.g. 1996; Meredith, 1993).

The effects of testing

Studies (Brown et al., 1999; Green & Ollerton, 1999) have identified students' anxiety about maths as a major issue in ITT. Additional prominence given to mathematical knowledge audits (DfEE 1998a) and tests for trainees may, paradoxically, prove counterproductive and jeopardise the achievements of training courses in improving students' attitude to mathematics (Brown et al., 1999). ITT has also been shown to be successful in increasing students' confidence in their ability to teach mathematics and shifting their absolutist beliefs (Bennett et al., 1993; Carre & Ernest, 1993; Carter et al., 1993; Brown et al., 1999). The significance of beliefs and conceptions on practice is well documented: teachers' dominant pedagogic beliefs are 'not inconsistent' with their dominant beliefs about the nature of mathematics (Andrews & Hatch, 1999); and, play a significant role in shaping teacher behaviours (Askew et al., 1997a; Lerman 1986, 1990; Ernest, 1989). Opinion is, however, divided as to how much ITT is able to substantively influence these beliefs. Primary B.Ed. students' images of teaching from their own school days have been shown to be highly influential in moulding subsequent classroom practice (Calderhead & Robson, 1991; n=12) and have necessitated much 'unlearning' in terms of attitude problems and subject misconceptions (Ball, 1988, 1990). Humanistic (Cheng, 1990; n=109) and pedagogic (Brown et al., 1999) views of teaching and learning developed in college sessions have also been found to be tempered by realism after teaching practice experiences. Additionally, ITT itself was not identified by serving teachers, or associated pupil outcome data, as a significant influence on the teaching of numeracy (Askew et al., 1997a; n=33). Bramald et al. (1995; n=162), however, argue that despite the perceived/reported lack of influence the effects of training courses were not constant and belief systems were not as resistant to change as some research suggested.

Intensity and prescription

ITT policy requirements can render courses over-full and squeeze out key aspects of training and professional development. Carre & Ernest (1993) expressed concern that an increasingly school based training would cause the already insignificant improvement in PGCE students' grasp of content, substantive and syntactic knowledge of mathematics to deteriorate further. Teachers perceived training to be too short and too rushed, and most did not consider it to be a significant feature of their professional development (Askew et al., 1997a). Yet ironically, in the European context, past British Governments have been on their own in attempting to erode both the length, and the university based academic rigour, of ITT (Holyoake, 1993). Identifying the more disposable components of training courses opinion suggests that 'reflection', universally popular in the 80's, 90's (MOTE, 1992), is a vehicle more appropriate for experienced teachers (McNally et al., 1994, 1997). ITT, it was felt, should prioritise preparation for the induction phase (McIntyre, 1993). There is little empirical evidence to suggest that the use of reflection in ITT is effective in connecting pedagogic theory with practice and, additionally, it was felt to be at odds with a competence based model of training. Critiques carry many health warnings

(McIntyre 1993; McNamara, 1990; Smith, 1991; Leat, 1995; Higgins & Leat, 1997; Tickle, 1994). Research suggests that reflective work, insofar as it exists can provide a forum in which students seek to reconstruct their own identity as they become inducted into professional discourses (e.g. Hanley & Brown, 1996, 1999; Tann, 1993; Jones et al., 2000).

Effectiveness of school/teacher led CPD

Mathematics Counts (DES, 1982) and HMI Reports (DES, 1978, 1979) generated considerable support for school-based professional development in the early 80s (Biggs, 1983; Pinner & Shuard, 1985; Pirie, 1987). This included links to the Mathematical Association Diploma in Mathematical Education (Melrose, 1982); projects such as LAMP (DES, 1987) and RAMP (Ahmed & Williams, 1991); and, 10/20 day courses (NFER). The use of advisory teachers (Straker, 1988; Biott, 1991) and mentoring of students and NQTs were identified as boosting the quality of the professional practice of the advisor/mentor as well as mentee (Vonk, 1993; Boydell, 1994; Elliott & Calderhead, 1993; Jaworski & Watson, 1994). Halpin (1990: 164), however, identified a lack of 'empirically or theoretically generalisable' evidence of the effectiveness of INSET. British studies focused specifically on the effectiveness of mathematics CPD, as regards pupil outcome data, are difficult to locate. Askew et al. (1997a, 1997b) identified extended mathematics programmes such as 10 and 20-day courses as the most effective way of changing beliefs and practices so as to significantly improve effectiveness in teaching numeracy. In reality, however, primary schools still retain an individualistic notion of development and short courses still predominate by virtue of time/cost constraints and perceived needs (Bottery & Wright, 1996).

Implications

Current government policies aimed at raising standards in primary schools have been experienced by many primary teachers in terms of initiative overload. Such policies can be seen as part of a perpetual readjustment in teaching styles, related to the evolution of learning theories and policy fashions (Brown, 1997). The implementation of the National Numeracy Strategy, together with its associated cascade-training programme, has been an unprecedented move towards a prescribed PCK and CPD for primary teachers. Initial indications are, however, that NNS has been well received with positive impacts reported upon teacher and pupil attitudes, and practices/outcomes respectively (Ofsted, 1998; McNamara et al., 2000). In the longer term, however, embedding policy has not always been understood in the terms in which it was presented (e.g. Millett, 1996), nor has it always been fully implemented before the next policy came along.

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