HOW DO WE TEACH CHILDREN TO BE NUMERATE?

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A Professional User Review of
UK research undertaken for the
British Educational Research Association

Contents

Introduction

This is a brief overview of the results of British research on primary numeracy for use by teachers and headteachers, policymakers and others who may be interested.

To structure the Professional User Review we have identified a number of themes that arise from the research literature. Under each theme the main arguments are first stated - as key messages from the research to users. These are followed by a short discussion that includes references to one or two key readings and leads to a box containing the implications of this research for teachers. The readings were chosen on the basis that the research reported has been accepted as valid and rigorous and, wherever possible the reports are British and relatively accessible.

We realise that there are some important gaps in coverage, both in curriculum areas like the teaching of fractions and in other areas like pupils' attitudes to mathematics. These are because we feel that there is not sufficient reliable, relevant and recent British research to include. We hope that research funding and appropriate staffing will allow some of these gaps to be filled as soon as possible.

The review is organized around the following themes:

- Early introduction to number
- From counting to number operations
- Mental calculation
- The importance of meaning
- Physical representations and mental imagery
- Calculators and computers
- Pedagogy
- Home cultures
- Teacher professional development
- Standards and government initiatives

Early introductions to number

Key Messages

- Pre-school children's experience of number is not always built upon when they come to school.
- Counting is an effective basis for the early years number curriculum.
- Young children can use idiosyncratic symbols to record small quantities but standard numerals are more helpful in solving problems.

The extent to which young children can benefit from the school's mathematics curriculum is influenced by their experience of maths and number in the years before they go to school. Aubrey (1997) investigated what children knew about number and found that their knowledge was related to their skill in reciting conventional counting sequences (rote counting). Children who could perform well on this were well on the way to National Curriculum level 1. However Aubrey concluded that children's rich experience of number was frequently ignored at school entry.

This may in part be due to the low status that is sometimes given to children's skill in rote counting. Traditionally the early years number curriculum was based on sorting and matching but it is now understood that the social functions of counting play an important role, including the kudos that children attach to 'being able to count'. While such counting may have no relationship with later skill in adding and subtracting, it does play an important role in providing children with access to talk about number.

British research into young children's use of number symbols has focused on their invention of idiosyncratic symbols. Using a game where children annotated tins to show how many bricks they contained Hughes (1986) found that even some pre-schoolers were able to represent small quantities. However, Munn (1994) found that when children used their own idiosyncratic notation they were less successful at solving simple problems (adding a brick) than those children who used conventional numerals.

Implications

- The knowledge that children bring to school needs to be built upon.
- Children need experience of counting in a variety of social contexts.
- Young children need to feel free to use a variety of ways, including conventional numerical symbols, to support simple problem solving.

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From counting to number operations

Key Messages
- There is a well-established sequence of development from counting into mental methods for addition and subtraction up to 20.
- To make progress children need to learn to compress counting procedures.

There is general agreement from a number of research studies that, for the operation of adding numbers up to 20, children progress through a sequence of: count all, count on from the first number, count on from the larger number, use known facts and derive number facts (Gray, 1991). There is also evidence that children can be taught to progress through this sequence. For example, teachers involved in a research project worked with low-attaining Year 3 children who were relying heavily on counting methods. The teachers identified those few number facts that these children did know (most often small doubles) and worked to help them derive unknown number facts. In an assessment after this intervention these children out-performed a control group with three times as many using known or derived facts (Askew, Bibby, & Brown, 2001).

In the case of lower attaining children there is a worry that over-dependence on counting for calculating may lead to their not committing number facts to memory. However, even children who know many number facts and have developed a range of calculation methods still sometimes combine these facts and methods with counting techniques in order to derive unknown facts (Thompson, 1995). Rather than try and encourage children to give up using counting techniques altogether, successful progression appears to rest on children learning to compress counting procedures, for example being able to count on in 2's starting from any even number or in 5's from any multiple of 5, adding, say, 7 to 38 possibly by partitioning the 7 into 2 and 5 and using the compressed counting on sequence 28, 30, 35.

Implications
- Children need to be encouraged to use more efficient counting processes.
- Some children need to be taught to develop links between known number facts and derived facts.

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Mental calculation

Key Messages

- Children use a variety of mental methods for calculating with numbers greater than 20.
- Children's effective mental strategies focus on partitioning multi-digit numbers in a variety of ways.
- Understanding the structure of number operations is essential for mental calculation strategies.

While there is general agreement on the order of development of strategies for adding numbers to 20, there is less agreement about strategies involving the addition and subtraction of numbers from 20 to 100. Some research (Denivr & Brown 1986) suggests that there is no uniques sequence, and that, moreover, there is no clear relationship between order of teaching and learning.

More recent research suggests two particularly common approaches (Thompson, 1999b). The first involves partitioning or splitting both numbers. For example,

47 + 36 is calculated as 40 + 30 = 70; 7 + 6 = 13; 70 + 13 = 83.

The second involves a sequencing or jump method:

47 + 36 calculated as 47 + 30 = 77; 77 + 6 = 83.

Studies carried out in Holland suggest that while children may tend to prefer to use the partitioning method, they should be encouraged to use the sequencing method as it lends itself more readily to subtraction (83 – 47 as 83 – 40 = 43; 43 – 7 = 36).

Scrutiny of such mental calculation strategies as used by children suggests that there is no evidence of what is normally understood by place value (tens and units) in their methods (Ruthven, 1998). Mental calculation strategies use what has been described as the quantity value aspect of place value (56 seen as 50 and 6), whereas standard written algorithms draw on the column value aspect (56 seen as 5 tens and 6 units) (Thompson, 1999a).

As well as understanding the structure of number in this quantitive way, children's understanding of the structure of number operations affects their mental strategies. Research shows that understanding the commutativity of number (a + b = b + a) is related to the use of more efficient computation strategies. Children's understanding of commutativity of multiplication develops later than that of addition and is also influenced by the type of problem (Nunes & Bryant, 1996).

Children's understanding of the inverse relation between addition and subtraction and of decomposition of numbers are closely related but these two are not related to knowledge of number facts. And while children are able to use their understanding of multiplication to solve division questions, they can do this much earlier that they are able to think of using division strategies to solve multiplication problems (Nunes, Schliemann, & Carraher, 1993)

Implications

- Given children's mental strategies, it makes sense to delay the teaching of algorithms that focus on a digit's column value.
- In developing mental strategies, teaching needs to attend to the structure of number operations as much as to the structure of numbers.

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The importance of meaning

Key Messages

- Understanding that each number operation can be associated with a variety of possible meanings is important for both calculation and application.
- Early meanings may limit later understandings.
- Careful use of language is key to developing the variety of meanings.

Calculations can be identified with several different types of interpretations and contextual problems. For example, 4 x 5 can be linked to:

- repeated sets (e.g. 4 boxes each with 5 hats);
- multiplicative comparison (scale factor) (e.g. 4 hats and 5 times as many scarves);
- rectangular arrays (e.g. 4 rows of 5 hats);
- Cartesian product (e.g. the number of different possibilities for wearing a hat and a scarf from 4 hats and 5 scarves).

Similarly division calculations can be interpreted in two ways. For example, 20 ÷ 5 can be associated with:

- measurement/grouping (quotition) (e.g. 20 apples put into bags of 5, how many bags get filled?);
- sharing (partitioning) (e.g. 20 apples put equally into 5 bags, how many apples in each bag?)

Of these possible interpretations, research has shown that multiplication as repeated addition and division as sharing appear to be widely understood by primary aged children. However, as the example above show, understanding the meaning of multiplication is more complex (Nunes & Bryant, 1996) and difficulties with fully understanding multiplication and division persist into secondary school (Hart, 1981).

There is evidence that such early ideas – multiplication as repeated addition and division as sharing – have an enduring effect and can limit children's later understandings of these operations. For example, understanding multiplication only as repeated addition may lead to misconceptions such as 'multiplication makes bigger' and 'division makes smaller' (Hart 1981, Greer 1988). Even with older children researchers have shown that they may persist with using primitive methods such as repeated addition or repeated subtraction with larger numbers (Anghileri, 1999).

Language is important here as different expressions will greatly influence children's solution methods. For example, interpreting 52 x 3 as '52 times 3' or '52 lots of 3' may lead to a less efficient calculation method than 'reading' the symbols as '52 multiplied by 3' or '3 fifty-twos'.

Implications

- Children need to have experience of the variety of meanings that can be associated with calculation sentences.
- They need to be encouraged to 'read' calculations in a variety of ways and to select the 'reading' that makes carrying out the calculation most efficient.

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Physical representations and mental imagery

Key Messages

- As children progress there are differences in the mental images used by low and high attainers.
- The empty number line provides a useful model for addition and subtraction, but it needs structured development.
- Standard written algorithms can provide efficient methods when they are understood but lead to errors if children are unable to reconstruct them.

Researchers investigating children's mental imagery for number and calculations have used children's verbal and written descriptions as a means of accessing this imagery. Lower attaining children describe images that suggest that they carry out mental procedures in ways that mirror how they would operate on tangible objects. As such, these children are limited in the mental procedures they draw upon. In contrast, higher attaining children show evidence of an implicit appreciation of the information compressed into mathematical symbolism and can draw on this to make choices over mental calculation methods.

Another study where children were asked to describe 'what was in their head' when they calculated showed the extent to which their mental images were influenced by the physical representations (verbal, pictorial, written or concrete) used by their teachers (Bills, 1999). This raises an important question about the most appropriate representations to use when teaching. The dominant tradition in the UK is to offer a wide range of representations. However, other cultures focus on a more limited range of representations.

For example, in the Netherlands, teachers draw on a few well-researched and evidence based representations such as the empty number line. The Dutch experience indicates that in order to be effective, the empty number line needs careful introduction and structured development: it cannot just be used occasionally to supplement other representations (Beishuizen, 1999). While the use of the empty number line has been widely advocated in England, only one research study has been reported (Rousham, 1997); this indicated that after some initial success in using this model to develop mental methods, most children reverted to formal methods within two months. This may, however, change as its introduction precedes formal methods.

But while formal methods and standard written algorithms provide efficient written methods when they are understood, they can often lead to errors when they are incompatible with informal approaches (Anghileri, 2001). Children tend to use algorithms as 'mechanical' procedures and where they do not understand the procedures the research evidence suggests that they are unable to reconstruct the processes involved.

Implications

- Children need to be encouraged to develop efficient mental images and such a range will be influenced by physical representations offered by the teacher.
- Working in a structured and systematic way with a limited but effective set ofrepresentations may be more helpful than offering children a wide range of representations.

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Calculators and computers

Key messages

- There is no research evidence that shows that the use of calculators in the classroom leads to poorer pupil performance.
- Significant beneficial effects on performance of using computers to teach numeracy are not yet supported by substantial research evidence.

In the 1980s there was a significant experiment in which clusters of schools implemented a Calculator-Aware Number (CAN) project , which emphasised calculator use and mental strategies without teaching any standard written methods. Comparisons of numeracy standards between the pupils involved in this project and control groups found either stronger performance among the CAN pupils or similar performances in both groups (Ruthven, 1998; Shuard, Walsh, Goodwin, & Worcester, 1991). This is consistent with international findings.

In the late 1990s, calculator use was widely blamed for perceived low numeracy standards. However there is no research evidence to support this; surveys showed that calculator use in primary schools in fact remained at very modest levels (School Curriculum and Assessment Authority (SCAA), 1997).

A study relating to classrooms where there was extensive use of information technology in numeracy teaching found short-term gains in performance in four out of five classes. However the researchers point out that the effects of ICT are difficult to isolate as the teachers involved also differed from their colleagues in other ways, for example in making greater use of collaborative work and less direct instruction or individual working (Mosely et al., 1999).

There have also been several studies evaluating the use of Integrated Learning Systems to teach numeracy. However the results have been inconsistent. There is some indication of better performance in basic skills (the main focus of these systems) and improved behaviour and attitudes, but not of improved performance in the type of numeracy reasoning tested in national tests (Underwood & Brown, 1997).

Implications

- Calculators could be used more frequently, in combination with mental methods, all through the primary school.
- More use of computers seems justified, especially for collaborative work but possibly also for basic skills consolidation, even though there is no evidence of large gains.

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Pedagogy

Key Messages
- There is no evidence that pupil gains are related to any one particular style of lesson organization.
- There is an association between teachers' beliefs about the teaching and learning of numeracy and pupil gains.
- Teachers' beliefs and pupils' perceptions relate to broader cultural expectations.

Pedagogy is a term that has no clearly agreed meaning. Many studies of pedagogy interpret this in broad terms considering factors such as the grouping of children, the layout of classrooms, use of resources and teachers' questioning styles. More recently some studies have turned to looking at teaching more closely and the teaching of particular subjects, mathematics included. This view of pedagogy is closer to what some would call the study of didactics.

Taking pedagogy in the broader sense, since the 1970's studies of primary teaching have been influenced by the 'traditional' versus 'progressive' debate. Findings from these early studies (Bennett, 1976; Galton & Simon, 1980) generally agreed that teacher questioning at a high cognitive level was a key factor in pupils' attainment. However, findings about lesson organisation that contributed to this were more ambivalent: while high-level questioning was often associated with higher proportions of whole class teaching this was by no means always the case.

More recently, a study of effective teachers of numeracy in primary schools (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997) examined pedagogy in terms of grouping, extent of whole class teaching and other aspects of classroom practice against pupil gains on a test of numeracy. No clear associations were identified between pupils gains on the test and such general aspects of pedagogy and further work by this team has confirmed this (Brown, 1999). However there was an association between pupil gains and their teachers' beliefs about how pupils learn and how best to teach numeracy. Teachers who both worked with their pupils' existing understandings and taught mathematics as a set of connected ideas had classes that made greater gains than either the group of teachers who put more emphasis on pupils' learning or the group of teachers who focused primarily on the act of teaching.

Studies making international comparisons have shown that teacher beliefs and pupils' perceptions relate to the broader cultural context within which mathematics lessons are located. For example, a study comparing primary mathematics in Japan and England noted two major differences. First, teachers in Japan placed more emphasis on effort and perseverance and regarded these as most important, whereas English teachers felt that 'innate' ability was the greatest influence and hence failed to challenge all children. Second, in Japan there was more attention paid to pupils working as a member of a group in ways that would maximize the chance of group success rather then promoting individual differences.

A study involving French and English classes found that the concern of English teachers to meet the needs of individuals meant that they placed a higher priority on making the work interesting in order to motivate pupils than the French teachers did. In France it appeared that the high societal value placed on intellectual endeavour meant that the French pupils displayed a clearer distinction between 'work' and 'play'

Implications

- Challenging pupils with high level cognitive questions may have more impact on standards than styles of lesson organization.
- Reasons given by teachers for low attainment such as 'lack of ability' may reflect more of the cultural beliefs than the reality of the situation.
- Standards are likely to rise if teachers place more emphasis on effort and perseverance and less on 'ability'.
- Teaching which works with pupils' existing understandings and connects up different ideas is likely to be more effective.

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Home cultures

Key Messages

- There are variations in performance amongst different ethnic and social class groupings that are much greater than those for gender.
- Home practices are linked both to cultural heritage and perceptions of what is most important for children to learn.

Although recent studies suggest that boys perform very slightly better than girls in tests of numeracy, the differences are small in comparison to those relating to social class (Brown & Millet, 2003). Social class differences are of the order of one year's academic progress, that is children from low socio-economic backgrounds are performing at a level about one year behind peers from higher socio-economic backgrounds. This is about the same margin as that separating the performances of different ethnic groups. Children of Chinese and Indian descent tend to perform on average as well as or better than European children, whereas African/Caribbean origin children do least well. There is some evidence that differences between ethnic groups are beginning to narrow. However the effects of class and ethnicity are not independent.

Studies have pointed to 'culture conflicts' in the ways that parents from different cultures and teachers view parents' roles in numeracy teaching, and in how they characterise desirable child behaviours (Jones, 1998).

But even within ethnic groups where cultural practices are similar, there can be quite large differences in the way that families structure home practices to support children's learning, for example the balance of emphasis between recall of tables and bonds, practice in 'school-type' books purchased by parents, and informal applications in home contexts (e.g. while shopping, cooking or playing games). These differences reflect which aspects of numeracy parents consider most important for their children to learn. One reason these differ, and may differ also from teachers' priorities, are the difficulties many parents experience in gaining access to and understanding approaches to numeracy used in their children's classrooms (Abreu, Cline, & Shamsi, 2001).

Implications

- Continued effort is needed to help some groups improve their attainment, especially children from lower socio-economic groups and/or of African/Caribbean origin.
- Many parents would appreciate more knowledge about the curriculum and teaching methods, and guidance as to how to help at home.

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Standards and government initiatives

Key Messages

- In international comparisons of attainment in primary numeracy, the UK countries have had below average scores.
- There is evidence of improving national standards, but it is difficult to separate out the effects of the National Numeracy Strategy and teaching to the test.
- Universal implementation of the National Numeracy Strategy has resulted in improvements in teaching and teacher confidence.

There have been two large-scale international comparisons of primary mathematics performance. In both of these the participating UK countries have scored below average overall, with relatively low scores in number topics (Harris, Keys, & Fernandes, 1997). Part of this problem may be explained by differences reflected in the test items in interpretations of numeracy. For example in the 1990s UK curricula put greater emphasis on applications, and less on written algorithms, compared to other countries.

Several sources of data concerning the effects of the National Numeracy Strategy suggest that this has helped to raise average standards of attainment. However there is some disagreement about the size of the effect, with the lowest estimates suggesting the equivalent of about 2 months' progress. The proportion of children obtaining level 4 in national tests at age 11 has risen steadily. However it is difficult to decide whether the Numeracy Strategy or the pressure on teachers to improve school performance in national tests has had a greater influence. The Strategy does not seem to have been effective in reducing the wide gap between the highest and lowest attainers. (Brown *et al.,* 2003)

A DfES commissioned evaluation of the implementation of the National Numeracy Strategy has demonstrated that virtually all teachers have changed their practices and believe that their own learning has been positively affected by the training provided (Earle et al., 2003). There is evidence that teaching has improved substantially.

However there is some doubt as to whether there has been 'deep change' in teachers' beliefs about teaching and learning of the sort that would fundamentally change the way teachers interact with children. Changes may be limited to adoptions of new lesson structures, new curricular emphases and new methods of presenting specific topics.

Implications

- Test scores can be raised by a combination of pressure on teachers and curriculum change, but there is some question about the extent of real improvement in learning.
- National training has helped to improve teacher confidence and teaching quality but it is harder to make fundamental changes in the way teachers interact with children.

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Teacher professional development

Key Messages

- Initial teacher education can be successful in improving students' attitudes to mathematics.
- Deeper understanding of the mathematics in the primary curriculum is more important for effective teaching than higher mathematical qualifications.

Several studies have shown that initial teacher education can be successful in increasing students' confidence in teaching mathematics. It can also make them more aware of the nature of mathematics as a human, social and changing creation incorporating different facets, rather than simply being a given set of procedures which have to be carried out correctly (Carter, Carre, & Bennett, 1993).

However opinion is divided as to how successful initial teacher education can be in changing fundamental beliefs about teaching and learning mathematics. For example, there is some evidence that more liberal attitudes that students developed during initial education were modified during classroom practice (Brown, McNamara, Hanley, & Jones, 1999).

In a study of effective teachers of primary numeracy (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997), it was found that teachers' beliefs about numeracy, and about numeracy teaching and learning, were strongly related to pupil gains made during the year. Effectiveness was related to connectedness of knowledge and beliefs, enabling teachers to relate their classroom practice to a variety of mathematical ideas, different mathematical representations, real life applications, and ways in which children learn.

This connectedness seemed to be related to experience of sustained professional development in the teaching of mathematics. This allowed time for reflection on practice with others who were more expert, either within or outside school. Higher mathematical qualifications did not relate to effectiveness, since they did not guarantee deeper understanding. However several studies suggest that lack of subject knowledge seems to be related to less effective teaching of mathematics or numeracy. This discrepancy may reflect the type of assessment used: measures of connected understanding seem to be the key.

Implications

- In order to be effective, initial teacher education and continuing professional development need to be linked, sustained and address broadening views about mathematics.
- A connected understanding of subject knowledge is important, together with links to applications, representations, classroom practices and children's learning.

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