more than WORDS

If it takes 10 minutes to read Mike Askew's article on problem solving, how much time will you save teaching maths this year?

Word problems get a lot of bad press in mathematics education. At best, they are seen as calculations wrapped up in words. Children learn to 'throw' away the context and 'strip out' the calculation. Trouble is, this can lead to nonsense such as children arguing that '3' must be the answer to the following problem:

If Henry the 8th had 6 wives, how many wives did Henry the 4th have?

Half the 8 to get 4, so half the 6!

Ironically this strategy works most of the time even if you don't understand the problem at all. I have a Chinese textbook for nine year olds. I don't understand a word of Mandarin, but the numbers are readable and I am confident that the problem involving 25 and 14 is most likely to be a multiplication, while the one with 3007 and 1896 is almost certainly a subtraction.

So are word problems just a hangover from the days when we thought children should be able to work out how long baths took to fill, or how many men were needed to dig holes? Or can they actually help children's understanding of mathematics? I think they can.

Learning from word problems

Word problems can help children's understanding if we think of them as a means of learning about calculation, rather than being introduced after arithmetic skills have been learned. The 'word' problem has to be treated as a more genuine challenge that children can use informal methods to solve, methods that the teacher can then help them 'craft' into more formal mathematics.

A group of American writers, led by Thomas Carpenter, have done extensive research into the sorts of problems that can provide 'root' situations for formal arithmetic. They've written a slim and easily read book setting out their findings, which I recommend to you if the following summary of their work appeals[1].

Roots of addition and subtraction

Carpenter and colleagues identify three roots of addition and subtraction: join or separate, part-part-whole and compare situations.

Join or separate

When two sets are put together to create a new set - or the reverse where a single set is split into two - are join or separate problems.

Seraphina put 15 tulips and 12 daffodils in an empty vase. How many flowers were in the vase? (Join sets of 15 and 12 to create a new set of 27.)

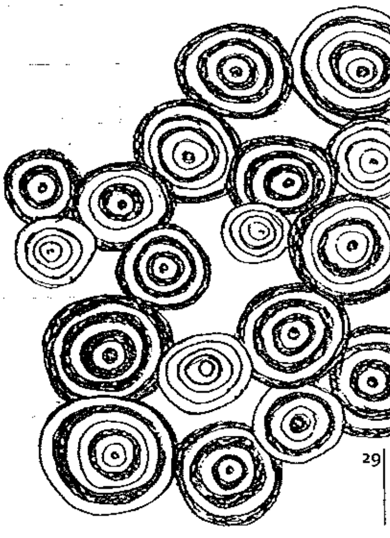
The next day 10 flowers had died. How many were left? (Separate 10 off from 27)

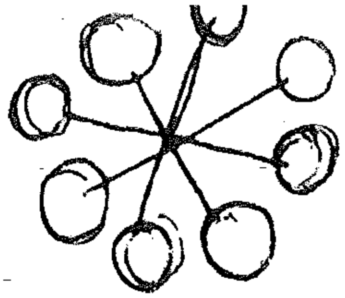
Children find join or separate problems the easiest to solve as the 'action' of the problem can be directly modelled. For example, you might put out 27 counters to represent the flowers and remove 10 of them. It is then quite a short step to show children that this might be recorded as 27 - 10.

Part-part-whole

Seraphina has 27 flowers in a vase. Fifteen of the flowers are tulips, how many are daffodils.

This is a typical part-part-whole problem. The whole and one of the parts are known, and the missing part has to be found. This is harder for children than the problem of putting the flowers into the vase because there is no 'action' in the story to model. Helping children model the 15 tulips with, say, counters and then counting on until 27 is reached takes skilful teacher intervention. Recording the problem as 15 + [ ] = 27 and modelling the solution on a number line helps to develop understanding.





Compare

Linda has 15 t-shirts, and Penny has 12. How many more t-shirts does Linda have? Here we need to compare 15 and 12 and find the difference. Compare problems pose children difficulties similar to part-part-whole due to the lack of action in the story.

As adults, we know that this can be written 15 – 12. However, children think of such written calculations as 'take-aways', not comparisons; after all, Linda and Penny still have the same number of t-shirts each at the start and end of the problem, nothing has been taken away.

Lots of talk about the informal solutions methods children use and introducing the formal notation over time will help connections to be gradually built up.

Roots of multiplication and division

Here are three main 'root' situations for multiplication: repeated addition, arrays and scaling. Linked to this are two models of division: grouping (repeated subtraction), and sharing.

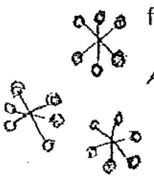
Repeated addition

The most familiar introduction to multiplication needs little explanation:

Su-Lin opened 5 packets of biscuits for the party and put them all on a plate. Each packet had 4 biscuits in it. How many biscuits were on the plate?
(4 + 4 + 4 + 4 + 4 or multiply 4 by 5)

Arrays

Array situations occur when objects are set out in a rectangular pattern (cakes in tins, blocks of stamps) and the total number has to be found.

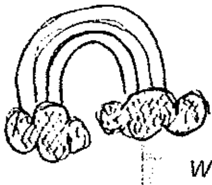


A sheet of stamps has 8 stamps in each row, and three rows. How many stamps are there on a sheet? (Multiply 8 by 3. Or multiply 3 by 8.)

Contexts involving arrays make the commutative property of multiplication (8 x 3 = 3 x 8) more explicit than contexts involving repeated subtraction. For example, thinking about opening 8 packets of 3 biscuits, children are more likely to calculate 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 than 8 + 8 + 8. But an array of cakes with 3 rows and 8 columns makes clear the possibility of either adding the rows or adding the columns.

Scaling

This is less common in primary schools. It involves situations where a continuous quantity is increased in size by a scaling factor.



When I got her, my pet carp was 6 cm long. She is now 4 times as long as that. How long is she now?
(6 multiplied by 4)

Grouping (repeated subtraction)

Situations where an amount has to be put into groups or portions of equal size, and the size of the group or portion is known in advance, provide an introduction to 'division as grouping' or 'division as repeated subtraction'.

I have 12 m of ribbon and I want to make hair ribbons that are each 2 m long. How many can I make?
(Repeatedly subtract 2 from 12 or divide 12 by 2.)

Division as sharing

Situations where an amount has to be put into equal sized groups and the number of groups or portions is known in advance are known as 'division as sharing'.

I have 12 m of ribbon and I want to make 6 hair ribbons that are all the same length. How long will each hair ribbon be?
(Share 12 amongst 6 or divide 12 by 6.)

In the next article I'll look further at how these root problems can be developed to challenge and extend children's thinking.

Create a silly problem

Give pairs of children slips of paper in three colours. On one they write a place (in the park, on the moon, etc), on a second a pair of characters (brother and sister, dentist and patient etc) and a calculation on the third slip (3x4, 100 – 25 or whatever is currently being worked on). Collect in and redistribute the papers so that every pair has a new place, pair of characters and a calculation. They use these to create a silly problem to act out or write up to share with the class.

[1] Thomas P. Carpenter et al (1999) Children's Mathematics: Cognitively Guided Instruction. Heinemann

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